SECTION 4.2 STUDY SET

VOCABULARY

Fill in the blanks.

1. To solve the system $\begin{cases} x = y + 1 \\ 3x + 2y = 8 \end{cases}$ using the method discussed in this section, we begin by **substituting** $y + 1$ for x in the second equation.

2. We say that the equation $y = 2x + 4$ is solved for **y**.

CONCEPTS

3. If the substitution method is used to solve $\begin{cases} x = y + 1 \\ 3x + 2y = 8 \end{cases}$ which equation should be used as the substitution equation? **$y = -3x$**

4. Suppose the substitution method will be used to solve $\begin{cases} x - 2y = 2 \\ 2x + 3y = 11 \end{cases}$. Find a substitution equation by solving one of the equations for one of the variables. **$x = 2 + 2y$**

5. Suppose $x - 4$ is substituted for y in the equation $x + 3y = 8$. Insert parentheses in $x + 3\mathbf{x} - 4 = 8$ to show the substitution.
$x + 3(x - 4) = 8$

6. Fill in the blank. With the substitution method, the objective is to use an appropriate substitution to obtain one equation in **one** variable.

7. A student uses the substitution method to solve the system $\begin{cases} 4a + 5b = 2 \\ b = 3a - 11 \end{cases}$ and finds that $a = 3$. What is the easiest way for her to determine the value of b?
Substitute 3 for *a* in the second equation.

8. a. Clear $\frac{x}{5} + \frac{2y}{3} = 1$ of fractions.
$3x + 10y = 15$
b. Write $2x + y = x - 5y + 3$ in the form $Ax + By = C$. **$x + 6y = 3$**

9. Suppose $-2 = 1$ is obtained when a system is solved by the substitution method.
a. Does the system have a solution? **No**
b. Which of the following is a possible graph of the system? **ii**

10. Suppose $2 = 2$ is obtained when a system is solved by the substitution method.
a. Does the system have a solution? **Yes**
b. Which graph is a possible graph of the system? **ii**

NOTATION

Complete the solution of the system.

11. Solve:
$\begin{cases} y = 3x \\ x - y = 4 \end{cases}$

$x - y = 4$ This is the second equation. Substitute for y.
$x - (\mathbf{3x}) = 4$
$-2x = \mathbf{4}$
$x = \mathbf{-2}$

$y = 3x$ This is the first equation.
$y = 3(\mathbf{-2})$
$y = \mathbf{-6}$

The solution is (**–2**, **–6**).

12. The system $\begin{cases} a = 3b + 2 \\ a + 3b = 8 \end{cases}$ was solved and it was found that b is 1 and a is 5. Write the solution as an ordered pair. **(5, 1)**

GUIDED PRACTICE

Solve each system by substitution.

13. $\begin{cases} y = 2x \\ x + y = 6 \end{cases}$

$x + y = 6$ The 2nd equation
Substitute for y.

$x + (\mathbf{2}x) = 6$
$3x = 6$
$\frac{3x}{\mathbf{3}} = \frac{6}{\mathbf{3}}$
$x = 2$

$y = 2x$ The 1st equation
$y = 2(\mathbf{2})$
$y = 4$

The solution is (2,4).

Check:	Check:
$y = 2x$	$x + y = 6$
$4 \stackrel{?}{=} 2(2)$	$2 + 4 \stackrel{?}{=} 6$
$4 = 4$	$6 = 6$
True	True

14, $\begin{cases} y = 3x \\ x + y = 4 \end{cases}$

$x + y = 4$ The 2nd equation
Substitute for y.

$x + (\mathbf{3}x) = 4$
$4x = 4$
$\frac{4x}{\mathbf{4}} = \frac{4}{\mathbf{4}}$
$x = 1$

$y = 3x$ The 1st equation
$y = 3(\mathbf{1})$
$y = 3$

The solution is (1, 3).

15. $\begin{cases} y = 2x - 6 \\ 2x + y = 6 \end{cases}$

$2x + y = 6$ The 2nd equation
Substitute for y.

$2x + (\mathbf{2}x - \mathbf{6}) = 6$
$4x - 6 = 6$
$4x - 6 + \mathbf{6} = 6 + \mathbf{6}$
$4x = 12$
$\frac{4x}{\mathbf{4}} = \frac{12}{\mathbf{4}}$
$x = 3$

$y = 2x - 6$ The 1st equation
$y = 2(\mathbf{3}) - 6$
$y = 6 - 6$
$y = 0$

The solution is (3, 0).

Check:	Check:
$y = 2x - 6$	$2x + y = 6$
$0 \stackrel{?}{=} 2(3) - 6$	$2(3) + 0 \stackrel{?}{=} 6$
$0 = 0$	$6 = 6$
True	True

16. $\begin{cases} y = 2x - 9 \\ x + 3y = 8 \end{cases}$

$x + 3y = 8$ The 2nd equation
Substitute for y.

$x + 3(\mathbf{2}x - \mathbf{9}) = 8$
$x + 6x - 27 = 8$
$7x - 27 = 8$
$7x - 27 + \mathbf{27} = 8 + \mathbf{27}$
$7x = 35$
$\frac{7x}{\mathbf{7}} = \frac{35}{\mathbf{7}}$
$x = 5$

$y = 2x - 9$ The 1st equation
$y = 2(\mathbf{5}) - 9$
$y = 10 - 9$
$y = 1$

The solution is (5, 1).

17. $\begin{cases} 3x+y=-4 \\ x=y \end{cases}$

$3x+y=-4$ The 1st equation
Substitute for x.
$3(\mathbf{y})+y=-4$
$4y=-4$
$\frac{4y}{\mathbf{4}}=\frac{-4}{\mathbf{4}}$
$y=-1$

$x=y$ The 2nd equation
$x=\mathbf{-1}$
The solution is $(-1,-1)$.

Check:
$3x+y=-4$
$3(-1)+(-1) \stackrel{?}{=} -4$
$-4=-4$
True

Check:
$x=y$
$-1 \stackrel{?}{=} -1$
$-1=-1$
True

18. $\begin{cases} x+2y=-6 \\ x=y \end{cases}$

$x+2y=-6$ The 1st equation
Substitute for x.
$\mathbf{y}+2y=-6$
$3y=-6$
$\frac{3y}{\mathbf{3}}=\frac{-6}{\mathbf{3}}$
$y=-2$

$x=y$ The 2nd equation
$x=\mathbf{-2}$
The solution is $(-2,-2)$.

19. $\begin{cases} x+3y=-4 \\ x=-5y \end{cases}$

$x+3y=-4$ The 1st equation
Substitute for x.
$-\mathbf{5y}+3y=-4$
$-2y=-4$
$\frac{-2y}{\mathbf{-2}}=\frac{-4}{\mathbf{-2}}$
$y=2$

$x=-5y$ The 2nd equation
$x=-5(\mathbf{2})$
$x=-10$
The solution is $(-10,2)$.

Check:
$x+3y=-4$
$-10+3(2) \stackrel{?}{=} -4$
$-4=-4$
True

Check:
$x=-5y$
$-10 \stackrel{?}{=} -5(2)$
$-10=-10$
True

20. $\begin{cases} x+5y=-3 \\ x=-4y \end{cases}$

$x+5y=-3$ The 1st equation
Substitute for x.
$-\mathbf{4y}+5y=-3$
$y=-3$

$x=-4y$ The 2nd equation
$x=-4(\mathbf{-3})$
$x=12$
The solution is $(12,-3)$.

21. $\begin{cases} 2x-y=-5 \\ x=-2y-5 \end{cases}$

$2x-y=-5$ The 1st equation
Substitute for x.
$2(-\mathbf{2y}-\mathbf{5})-y=-5$
$-4y-10-y=-5$
$-5y-10=-5$
$-5y-10+\mathbf{10}=-5+\mathbf{10}$
$-5y=5$
$\frac{-5y}{\mathbf{-5}}=\frac{5}{\mathbf{-5}}$
$y=-1$

$x=-2y-5$ The 2nd equation
$x=-2(\mathbf{-1})-5$
$x=2-5$
$x=-3$
The solution is $(-3,-1)$.

Check:
$2x-y=-5$
$2(-3)-(-1) \stackrel{?}{=} -5$
$-6+1 \stackrel{?}{=} -5$
$-5=-5$
True

Check:
$x=-2y-5$
$-3 \stackrel{?}{=} -2(-1)-5$
$-3 \stackrel{?}{=} 2-5$
$-3=-3$
True

22. $\begin{cases} y = -2x \\ 3x + 2y = -1 \end{cases}$

$3x + 2y = -1$ The 2nd equation
Substitute for y.

$3x + 2(-\mathbf{2}x) = -1$
$3x - 2x = -1$
$-x = -1$
$\frac{-x}{-\mathbf{1}} = \frac{-1}{-\mathbf{1}}$
$x = 1$

$y = -2x$ The 1st equation
$y = -2(\mathbf{1})$
$y = -2$
The solution is $(1, -2)$.

23. $\begin{cases} b = \frac{2}{3}a \\ 8a - 3b = 3 \end{cases}$

$8a - 3b = 3$ The 2nd equation
Substitute for b.

$8a - 3\left(\frac{\mathbf{2}}{\mathbf{3}}\boldsymbol{a}\right) = 3$
$8a - 2a = 3$
$6a = 3$
$\frac{6a}{\mathbf{6}} = \frac{3}{\mathbf{6}}$
$a = \frac{1}{2}$

$b = \frac{2}{3}a$ The 1st equation
$b = \frac{2}{3}\left(\frac{\mathbf{1}}{\mathbf{2}}\right)$
$b = \frac{1}{3}$

The solution is $\left(\frac{1}{2}, \frac{1}{3}\right)$.

Check:
$b = \frac{2}{3}a$
$\frac{1}{3} \stackrel{?}{=} \frac{2}{3}\left(\frac{1}{2}\right)$
$\frac{1}{3} = \frac{1}{3}$
True

Check:
$8a - 3b = 3$
$8\left(\frac{1}{2}\right) - 3\left(\frac{1}{3}\right) \stackrel{?}{=} 3$
$4 - 1 \stackrel{?}{=} 3$
$3 = 3$
True

24. $\begin{cases} a = \frac{2}{3}b \\ 9a + 4b = 5 \end{cases}$

$9a + 4b = 5$ The 2nd equation
Substitute for b.

$9\left(\frac{\mathbf{2}}{\mathbf{3}}\boldsymbol{b}\right) + 4b = 5$
$6b + 4b = 5$
$10b = 5$
$\frac{10b}{\mathbf{10}} = \frac{5}{\mathbf{10}}$
$b = \frac{1}{2}$

$a = \frac{2}{3}b$ The 1st equation
$a = \frac{2}{3}\left(\frac{\mathbf{1}}{\mathbf{2}}\right)$
$a = \frac{1}{3}$

The solution is $\left(\frac{1}{3}, \frac{1}{2}\right)$.

25. $\begin{cases} 2x+5y=-2 \\ y=-\dfrac{x}{2} \end{cases}$

Clear the fraction from the 2nd equation.

$\begin{cases} 2x+5y=-2 \\ -2y=x \end{cases}$

$2x+5y=-2$ The 1st equation
Substitute for x.

$2(-\mathbf{2}y)+5y=-2$
$-4y+5y=-2$
$y=-2$

$y=-\dfrac{x}{2}$ The 2nd equation

$-\mathbf{2}=-\dfrac{x}{2}$

$-2(-2)=-2\left(-\dfrac{x}{2}\right)$

$4=x$

The solution is $(4,-2)$.

Check:
$2x+5y=-2$
$2(4)+5(-2)\stackrel{?}{=}-2$
$8-10\stackrel{?}{=}-2$
$-2=-2$
True

Check:
$y=-\dfrac{x}{2}$
$-2\stackrel{?}{=}-\left(\dfrac{4}{2}\right)$
$-2=-2$
True

26. $\begin{cases} y=-\dfrac{x}{2} \\ 2x-3y=-7 \end{cases}$

Clear the fraction from the 1st equation.

$\begin{cases} -2y=x \\ 2x-3y=-7 \end{cases}$

$2x-3y=-7$ The 2nd equation
Substitute for x.

$2(-\mathbf{2}y)\text{-}3y=-7$
$-4y-3y=-7$
$-7y=-7$
$\dfrac{-7y}{-\mathbf{7}}=\dfrac{-7}{-\mathbf{7}}$
$y=1$

$-2y=x$ The 1st equation
$-2(\mathbf{1})=x$
$-2=x$

The solution is $(-2,1)$.

27. $\begin{cases} x=\dfrac{1}{3}y-1 \\ x=y+5 \end{cases}$

Clear the fraction from the 1st equation.

$\begin{cases} 3x=y-3 \\ x=y+5 \end{cases}$

$3x=y-3$ The 1st equation
Substitute for x.

$3(\boldsymbol{y}+\mathbf{5})=y-3$
$3y+15=y-3$
$3y+15-\mathbf{15}=y-3-\mathbf{15}$
$3y=y-18$
$3y-\boldsymbol{y}=y-18-\boldsymbol{y}$
$2y=-18$
$\dfrac{2y}{\mathbf{2}}=\dfrac{-18}{\mathbf{2}}$
$y=-9$

$x=y+5$ The 2nd equation
$x=-\mathbf{9}+5$
$x=-4$

The solution is $(-4,-9)$.

Check:
$x=\dfrac{1}{3}y-1$
$-4\stackrel{?}{=}\dfrac{1}{3}(-9)-1$
$-4\stackrel{?}{=}-3-1$
$-4=-4$
True

Check:
$x=y+5$
$-4\stackrel{?}{=}-9+5$
$-4=-4$
True

28. $\begin{cases} x = \frac{1}{2}y + 2 \\ x = y - 6 \end{cases}$

$x = \frac{1}{2}y + 2$ The 1st equation
Substitute for x.

$\boldsymbol{y} - \mathbf{6} = \frac{1}{2}y + 2$

$2(y) - 2(6) = 2\left(\frac{1}{2}y\right) + 2(2)$

$2y - 12 = y + 4$

$2y - 12 + \mathbf{12} = y + 4 + \mathbf{12}$

$2y = y + 16$

$2y - \boldsymbol{y} = y + 16 - \boldsymbol{y}$

$y = 16$

$x = y - 6$ The 2nd equation

$x = \mathbf{16} - 6$

$x = 10$

The solution is (10, 16).

Solve each system by substitution.

29. $\begin{cases} r + 3s = 9 \\ 3r + 2s = 13 \end{cases}$

Solve for r in the 1st equation.

$\begin{cases} r = 9 - 3s \\ 3r + 2s = 13 \end{cases}$

$3r + 2s = 13$ The 2nd equation
Substitute for x.

$3(\mathbf{9} - \mathbf{3}\boldsymbol{s}) + 2s = 13$

$27 - 9s + 2s = 13$

$27 - 7s = 13$

$27 - 7s - \mathbf{27} = 13 - \mathbf{27}$

$-7s = -14$

$\frac{-7s}{\mathbf{-7}} = \frac{-14}{\mathbf{-7}}$

$s = 2$

$r = 9 - 3s$ The 1st equation

$r = 9 - 3(\mathbf{2})$

$r = 9 - 6$

$r = 3$

The solution is (3, 2).

Check:

$r + 3s = 9$

$3 + 3(2) \stackrel{?}{=} 9$

$3 + 6 \stackrel{?}{=} 9$

$9 = 9$

True

Check:

$3r + 2s = 13$

$3(3) + 2(2) \stackrel{?}{=} 13$

$9 + 4 \stackrel{?}{=} 13$

$13 = 13$

True

30. $\begin{cases} x - 2y = 2 \\ 2x + 3y = 11 \end{cases}$

Solve for x in the 1st equation.

$\begin{cases} x = 2 + 2y \\ 2x + 3y = 11 \end{cases}$

$2x + 3y = 11$ The 2nd equation
Substitute for x.

$2(\mathbf{2} + \mathbf{2}\boldsymbol{y}) + 3y = 11$

$4 + 4y + 3y = 11$

$4 + 7y = 11$

$4 + 7y - \mathbf{4} = 11 - \mathbf{4}$

$7y = 7$

$\frac{7y}{\mathbf{7}} = \frac{7}{\mathbf{7}}$

$y = 1$

$x = 2 + 2y$ The 1st equation

$x = 2 + 2(\mathbf{1})$

$x = 2 + 2$

$x = 4$

The solution is (4, 1).

31. $\begin{cases} 4x+y=-15 \\ 2x+3y=5 \end{cases}$

Solve for y in the 1st equation.

$\begin{cases} y=-4x-15 \\ 2x+3y=5 \end{cases}$

$2x+3y=5$ The 2nd equation
Substitute for y.

$2x+3(-\mathbf{4}x-\mathbf{15})=5$
$2x-12x-45=5$
$-10x-45=5$
$-10x-45+\mathbf{45}=5+\mathbf{45}$
$-10x=50$
$\frac{-10x}{\mathbf{-10}}=\frac{50}{\mathbf{-10}}$
$x=-5$

$y=-4x-15$ The 1st equation
$y=-4(-\mathbf{5})-15$
$y=20-15$
$y=5$

The solution is $(-5, 5)$.

Check:
$4x+y=-15$
$4(-5)+5\stackrel{?}{=}-15$
$-20+5\stackrel{?}{=}-15$
$-15=-15$
True

Check:
$2x+3y=5$
$2(-5)+3(5)\stackrel{?}{=}5$
$-10+15\stackrel{?}{=}5$
$5=5$
True

32. $\begin{cases} 4x+y=-5 \\ 2x-3y=-13 \end{cases}$

Solve for y in the 1st equation.

$\begin{cases} y=-4x-5 \\ 2x-3y=-13 \end{cases}$

$2x-3y=-13$ The 2nd equation
Substitute for y.

$2x-3(-\mathbf{4}x-\mathbf{5})=-13$
$2x+12x+15=-13$
$14x+15=-13$
$14x+15-\mathbf{15}=-13-\mathbf{15}$
$14x=-28$
$\frac{14x}{\mathbf{14}}=\frac{-28}{\mathbf{14}}$
$x=-2$

$y=-4x-5$ The 1st equation
$y=-4(-\mathbf{2})-5$
$y=8-5$
$y=3$

The solution is $(-2, 3)$.

33. $\begin{cases} 6x-3y=5 \\ 2y+x=0 \end{cases}$

Solve for x in the 2nd equation.

$\begin{cases} 6x-3y=5 \\ x=-2y \end{cases}$

$6x-3y=5$ The 1st equation
Substitute for x.

$6(-\mathbf{2}y)-3y=5$
$-12y-3y=5$
$-15y=5$
$\frac{-15y}{\mathbf{-15}}=\frac{5}{\mathbf{-15}}$
$y=-\frac{1}{3}$

$x=-2y$ The 2nd equation
$x=-2\left(-\frac{\mathbf{1}}{\mathbf{3}}\right)$
$x=\frac{2}{3}$

The solution is $\left(\frac{2}{3}, -\frac{1}{3}\right)$.

Check:
$6x-3y=5$
$6\left(\frac{2}{3}\right)-3\left(-\frac{1}{3}\right)\stackrel{?}{=}5$
$4+1\stackrel{?}{=}5$
$5=5$
True

Check:
$x+2y=0$
$\frac{2}{3}+2\left(-\frac{1}{3}\right)\stackrel{?}{=}0$
$\frac{2}{3}-\frac{2}{3}\stackrel{?}{=}0$
$0=0$
True

34. $\begin{cases} 5s+10t=3 \\ 2s+t=0 \end{cases}$

Solve for t in the 2nd equation.

$\begin{cases} 5s+10t=3 \\ t=-2s \end{cases}$

$5s+10t=3$ The 1st equation
Substitute for t.

$5s+10(-\mathbf{2}\boldsymbol{s})=3$
$5s-20s=3$
$-15s=3$
$\frac{-15s}{-\mathbf{15}}=\frac{3}{-\mathbf{15}}$
$s=-\frac{1}{5}$

$t=-8s$ The 2nd equation
$t=-2\left(-\frac{\mathbf{1}}{\mathbf{5}}\right)$
$t=\frac{1}{5}$

The solution is $\left(-\frac{1}{5}, \frac{2}{5}\right)$.

35. $\begin{cases} 2x+3=-4y \\ x-6=-8y \end{cases}$

Solve for x in 2nd equation.

$\begin{cases} 2x+3=-4y \\ x=6-8y \end{cases}$

$2x+3=-4y$ The 1st equation
Substitute for x.

$2(\mathbf{6}-\mathbf{8}\boldsymbol{y})+3=-4y$
$12-16y+3=-4y$
$-16y+15=-4y$
$-16y+15+\mathbf{16}\boldsymbol{y}=-4y+\mathbf{16}\boldsymbol{y}$
$15=12y$
$\frac{15}{\mathbf{12}}=\frac{12y}{\mathbf{12}}$
$\frac{5}{4}=y$

$x=6-8y$ The 2nd equation
$x=6-8\left(\frac{\mathbf{5}}{\mathbf{4}}\right)$
$x=6-10$
$x=-4$

The solution is $\left(-4, \frac{5}{4}\right)$.

Check:

$2x+3=-4y$
$2(-4)+3\overset{?}{=}-4\left(\frac{5}{4}\right)$
$-8+3\overset{?}{=}-5$
$-5=-5$
True

Check:

$x-6=-8y$
$-4-6\overset{?}{=}-8\left(\frac{5}{4}\right)$
$-10=-10$
True

36. $\begin{cases} 5y+2=-4x \\ x+2y=-2 \end{cases}$

Solve for x in the 2nd equation.

$\begin{cases} 5y+2=-4x \\ x=-2-2y \end{cases}$

$5y+2=-4x$ The 1st equation
Substitute for x.

$5y+2=-4(-\mathbf{2}-\mathbf{2}\boldsymbol{y})$
$5y+2=8+8y$
$5y+2-\mathbf{8}=8+8y-\mathbf{8}$
$5y-6=8y$
$5y-6-\mathbf{5}\boldsymbol{y}=8y-\mathbf{5}\boldsymbol{y}$
$-6=3y$
$\frac{-6}{\mathbf{3}}=\frac{3y}{\mathbf{3}}$
$-2=y$

$x=-2-2y$ The 2nd equation
$x=-2-2(-\mathbf{2})$
$x=-2+4$
$x=2$

The solution is $(2,-2)$.

37. $\begin{cases} 2a-3b=-13 \\ -b=-2a-7 \end{cases}$

Solve for b in the 2nd equation.

$\begin{cases} 2a-3b=-13 \\ b=2a+7 \end{cases}$

$2a-3b=-13$ The 1st equation
Substitute for b.

$2a-3(\mathbf{2}a+\mathbf{7})=-13$
$2a-6a-21=-13$
$-4a-21=-13$
$-4a-21+\mathbf{21}=-13+\mathbf{21}$
$-4a=8$
$\frac{-4a}{-\mathbf{4}}=\frac{8}{-\mathbf{4}}$
$a=-2$

$b=2a+7$ The 2nd equation
$b=2(-\mathbf{2})+7$
$b=-4+7$
$b=3$

The solution is $(-2, 3)$.

Check:

$2a-3b=-13$
$2(-2)-3(3)\overset{?}{=}-13$
$-4-9\overset{?}{=}-13$
$-13=-13$
True

Check:

$-b=-2a-7$
$-(3)\overset{?}{=}-2(-2)-7$
$-3\overset{?}{=}4-7$
$-3=-3$
True

38. $\begin{cases} a-3b=-1 \\ -b=-2a-2 \end{cases}$

Solve for b in the 2nd equation.

$\begin{cases} a-3b=-1 \\ b=2a+2 \end{cases}$

$a-3b=-1$ The 1st equation
Substitute for b

$a-3(\mathbf{2}a+\mathbf{2})=-1$
$a-6a-6=-1$
$-5a-6=-1$
$-5a-6+\mathbf{6}=-1+\mathbf{6}$
$-5a=5$
$\frac{-5a}{-\mathbf{5}}=\frac{5}{-\mathbf{5}}$
$a=-1$

$b=2a+2$ The 2nd equation
$b=2(-\mathbf{1})+2$
$b=-2+2$
$b=0$

The solution is $(-1, 0)$.

39. $\begin{cases} 8x-6y=4 \\ 2x-y=-2 \end{cases}$

Solve for y in the 2nd equation.

$\begin{cases} 8x-6y=4 \\ y=2x+2 \end{cases}$

$8x-6y=4$ The 1st equation
Substitute for y.

$8x-6(\mathbf{2}x+\mathbf{2})=4$
$8x-12x-12=4$
$-4x-12=4$
$-4x-12+\mathbf{12}=4+\mathbf{12}$
$-4x=16$
$\frac{-4x}{-\mathbf{4}}=\frac{16}{-\mathbf{4}}$
$x=-4$

$y=2x+2$ The 2nd equation
$y=2(-\mathbf{4})+2$
$y=-8+2$
$y=-6$

The solution is $(-4, -6)$.

Check:

$8x-6y=4$
$8(-\mathbf{4})-6(-\mathbf{6})\overset{?}{=}4$
$-32+36\overset{?}{=}4$
$4=4$
True

Check:

$2x-y=-2$
$2(-4)-(-6)\overset{?}{=}-2$
$-8+6\overset{?}{=}-2$
$-2=-2$
True

40. $\begin{cases} 5x+4y=0 \\ 2x-y=0 \end{cases}$

Solve for y in the 2nd equation.

$\begin{cases} 5x+4y=0 \\ y=2x \end{cases}$

$5x+4y=0$ The 1st equation
Substitute for y.

$5x+4(\mathbf{2}x)=0$
$5x+8x=0$
$13x=0$
$\frac{13x}{\mathbf{13}}=\frac{0}{\mathbf{13}}$
$x=0$

$y=2x$ The 2nd equation
$y=2(\mathbf{0})$
$y=0$

The solution is (0, 0).

41. $\begin{cases} 4x+5y=2 \\ 3x-y=11 \end{cases}$

Solve for y in the 2nd equation.

$\begin{cases} 4x+5y=2 \\ y=3x-11 \end{cases}$

$4x+5y=2$ The 1st equation
Substitute for y.

$4x+5(\mathbf{3}x-\mathbf{11})=2$
$4x+15x-55=2$
$19x-55=2$
$19x-55+\mathbf{55}=2+\mathbf{55}$
$19x=57$
$\frac{19x}{\mathbf{19}}=\frac{57}{\mathbf{19}}$
$x=3$

$y=3x-11$ The 2nd equation
$y=3(\mathbf{3})-11$
$y=9-11$
$y=-2$

The solution is $(3,-2)$.

Check:

$4x+5y=2$
$4(3)+5(-2)\overset{?}{=}2$
$12-10\overset{?}{=}2$
$2=2$
True

Check:

$3x-y=11$
$3(3)-(-2)\overset{?}{=}11$
$9+2\overset{?}{=}11$
$11=11$
True

42. $\begin{cases} 5u+3v=5 \\ 4u-v=4 \end{cases}$

Solve for v in the 2nd equation.

$\begin{cases} 5u+3v=5 \\ v=4u-4 \end{cases}$

$5u+3v=5$ The 1st equation
Substitute for v.

$5u+3(\mathbf{4}u-\mathbf{4})=5$
$5u+12u-12=5$
$17u-12+\mathbf{12}=5+\mathbf{12}$
$17u=17$
$\frac{17u}{\mathbf{17}}=\frac{17}{\mathbf{17}}$
$u=1$

$v=4u-4$ The 2nd equation
$v=4(\mathbf{1})-4$
$v=4-4$
$v=0$

The solution is (1,0).

43. $\begin{cases} 3x+4y=-19 \\ 2y-x=3 \end{cases}$

Solve for x in the 2nd equation.

$\begin{cases} 3x+4y=-19 \\ x=2y-3 \end{cases}$

$3x+4y=-19$ The 1st equation
Substitute for x.

$3(\mathbf{2}y-\mathbf{3})+4y=-19$
$6y-9+4y=-19$
$10y-9=-19$
$10y-9+\mathbf{9}=-19+\mathbf{9}$
$10y=-10$
$\frac{10y}{\mathbf{10}}=\frac{-10}{\mathbf{10}}$
$y=-1$

$x=2y-3$ The 2nd equation
$x=2(-\mathbf{1})-3$
$x=-2-3$
$x=-5$

The solution is $(-5,-1)$.

Check:
$3x+4y=-19$
$3(-5)+4(-1)\overset{?}{=}-19$
$-15-4\overset{?}{=}-19$
$-19=-19$
True

Check:
$2y-x=3$
$2(-1)-(-5)\overset{?}{=}3$
$-2+5\overset{?}{=}3$
$3=3$
True

44. $\begin{cases} 5x-2y=-7 \\ 5-y=-3x \end{cases}$

Solve for y in the 2nd equation.

$\begin{cases} 5x-2y=-7 \\ y=3x+5 \end{cases}$

$5x-2y=-7$ The 1st equation
Substitute for y.

$5x-2(\mathbf{3x+5})=-7$
$5x-6x-10=-7$
$-x-10=-7$
$-x-10+\mathbf{10}=-7+\mathbf{10}$
$-x=3$
$\frac{-x}{-\mathbf{1}}=\frac{3}{-\mathbf{1}}$
$x=-3$

$y=3x+5$ The 2nd equation
$y=3(-\mathbf{3})+5$
$y=-9+5$
$y=-4$

The solution is $(-3,-4)$.

Solve each system by substituion.

45. $\begin{cases} \frac{x}{2}+\frac{y}{2}=-1 \\ \frac{x}{3}-\frac{y}{2}=-4 \end{cases}$

Clear both equations of fractions.

$\begin{cases} 2\left(\frac{x}{2}\right)+2\left(\frac{y}{2}\right)=2(-1) \\ 6\left(\frac{x}{3}\right)-6\left(\frac{y}{2}\right)=6(-4) \end{cases}$

$\begin{cases} x+y=-2 \\ 2x-3y=-24 \end{cases}$

Solve for x in the 1st equation.

$\begin{cases} x=-2-y \\ 2x-3y=-24 \end{cases}$

$2x-3y=-24$ The 2nd equation
Substitute for x.

$2(-\mathbf{2}-\boldsymbol{y})-3y=-24$
$-4-2y-3y=-24$
$-5y-4=-24$
$-5y-4+\mathbf{4}=-24+\mathbf{4}$
$-5y=-20$
$\frac{-5y}{-\mathbf{5}}=\frac{-20}{-\mathbf{5}}$
$y=4$

$x=-2-y$ The 1st equation
$x=-2-4$
$x=-6$

The solution is $(-6, 4)$.

Check:
$\frac{x}{2}+\frac{y}{2}=-1$
$\frac{-6}{2}+\frac{4}{2}\overset{?}{=}-1$
$-3+2\overset{?}{=}-1$
$-1=-1$
True

Check:
$\frac{x}{3}-\frac{y}{2}=-4$
$\frac{-6}{3}-\frac{4}{2}\overset{?}{=}-4$
$-2-2\overset{?}{=}-4$
$-4=-4$
True

46. $\begin{cases} \frac{2}{3}a+\frac{b}{5}=1 \\ \frac{a}{3}-\frac{2}{3}b=\frac{13}{3} \end{cases}$

Clear both equations of fractions.

$$\begin{cases} 15\left(\frac{2}{3}a\right)+15\left(\frac{b}{5}\right)=15(1) \\ 3\left(\frac{a}{3}\right)-3\left(\frac{2}{3}b\right)=3\left(\frac{13}{3}\right) \end{cases}$$

$$\begin{cases} 10a+3b=15 \\ a-2b=13 \end{cases}$$

Solve for a in the 2nd equation.

$$\begin{cases} 10a+3b=15 \\ a=2b+13 \end{cases}$$

$10a+3b=15$ The 1st equation Substitute for a.

$$10(\mathbf{2}b+\mathbf{13})+3b=15$$
$$20b+130+3b=15$$
$$23b+130=15$$
$$23b+130-\mathbf{130}=15-\mathbf{130}$$
$$23b=-115$$
$$\frac{23b}{\mathbf{23}}=\frac{-115}{\mathbf{23}}$$
$$b=-5$$

$a=2b+13$ The 2nd equation

$$a=2(-\mathbf{5})+13$$
$$a=-10+13$$
$$a=3$$

The solution is $(3,-5)$.

47. $\begin{cases} 5x=\frac{1}{2}y-1 \\ \frac{1}{4}y=10x-1 \end{cases}$

Clear both equations of fractions.

$$\begin{cases} 2(5x)=2\left(\frac{1}{2}y\right)-2(1) \\ 4\left(\frac{1}{4}y\right)=4(10x)-4(1) \end{cases}$$

$$\begin{cases} 10x=y-2 \\ y=40x-4 \end{cases}$$

$10x=y-2$ The 1st equation Substitute for y.

$$10x=(\mathbf{40}x-\mathbf{4})-2$$
$$10x=40x-6$$
$$10x-\mathbf{40}x=40x-6-\mathbf{40}x$$
$$-30x=-6$$
$$\frac{-30x}{\mathbf{-30}}=\frac{-6}{\mathbf{-30}}$$
$$x=\frac{1}{5}$$

$y=40x-4$ The 2st equation

$$y=40\left(\frac{\mathbf{1}}{\mathbf{5}}\right)-4$$
$$y=8-4$$
$$y=4$$

The solution is $\left(\frac{1}{5},4\right)$

Check:

$$5x=\frac{1}{2}y-1$$
$$5\left(\frac{1}{5}\right)\overset{?}{=}\frac{1}{2}(4)-1$$
$$1\overset{?}{=}2-1$$
$$1=1$$
True

Check:

$$\frac{1}{4}y=10x-1$$
$$\frac{1}{4}(4)\overset{?}{=}10\left(\frac{1}{5}\right)-1$$
$$1\overset{?}{=}2-1$$
$$1=1$$
True

48. $\begin{cases} \frac{x}{4}+y=\frac{1}{4} \\ \frac{y}{2}+\frac{11}{20}=\frac{x}{10} \end{cases}$

Clear both equations of fractions.

$$\begin{cases} 4\left(\frac{x}{4}\right)+4(y)=4\left(\frac{1}{4}\right) \\ 20\left(\frac{y}{2}\right)+20\left(\frac{11}{20}\right)=20\left(\frac{x}{10}\right) \end{cases}$$

$$\begin{cases} x+4y=1 \\ 10y+11=2x \end{cases}$$

Solve for x in the 1st equation.

$$\begin{cases} x=1-4y \\ 10y+11=2x \end{cases}$$

$10y+11=2x$ The 2nd equation
Substitute for x.

$$10y+11=2(\mathbf{1}-\mathbf{4}y)$$
$$10y+11=2-8y$$
$$10y+11-11=2-8y-11$$
$$10y=-9-8y$$
$$10y+\mathbf{8}y=-9-8y+\mathbf{8}y$$
$$18y=-9$$
$$\frac{18y}{\mathbf{18}}=\frac{-9}{\mathbf{18}}$$
$$y=-\frac{1}{2}$$

$x=1-4y$ The 1st equation

$$x=1-4\left(-\frac{\mathbf{1}}{\mathbf{2}}\right)$$
$$x=1+2$$
$$x=3$$

The solution is $\left(3,-\frac{1}{2}\right)$.

49. $\begin{cases} x-\frac{4}{5}y=4 \\ \frac{y}{3}=\frac{x}{2}-\frac{5}{2} \end{cases}$

Clear both equations of fractions.

$$\begin{cases} 5(x)-5\left(\frac{4}{5}y\right)=5(4) \\ 6\left(\frac{y}{3}\right)=6\left(\frac{x}{2}\right)-6\left(\frac{5}{2}\right) \end{cases}$$

$$\begin{cases} 5x-4y=20 \\ 2y=3x-15 \end{cases}$$

Solve for y in the 2nd equation.

$$\begin{cases} 5x-4y=20 \\ y=\frac{3}{2}x-\frac{15}{2} \end{cases}$$

$5x-4y=20$ The 1st equation
Substitute for y.

$$5x-4\left(\frac{\mathbf{3}}{\mathbf{2}}\boldsymbol{x}-\frac{\mathbf{15}}{\mathbf{2}}\right)=20$$
$$5x-6x+30=20$$
$$-x+30=20$$
$$-x+30-\mathbf{30}=20-\mathbf{30}$$
$$-x=-10$$
$$\frac{-x}{-\mathbf{1}}=\frac{-10}{-\mathbf{1}}$$
$$x=10$$

$y=\frac{3}{2}x-\frac{15}{2}$ The 2nd equation

$$y=\frac{3}{2}(\mathbf{10})-\frac{15}{2}$$
$$y=\frac{30}{2}-\frac{15}{2}$$
$$y=\frac{15}{2}$$

The solution is $\left(10,\frac{15}{2}\right)$.

Check:

$$x-\frac{4y}{5}=4$$
$$10-\frac{4(7.5)}{5}\stackrel{?}{=}4$$
$$10-6\stackrel{?}{=}4$$
$$4=4$$
True

Check:

$$\frac{y}{3}=\frac{x}{2}-\frac{5}{2}$$
$$\frac{7.5}{3}\stackrel{?}{=}\frac{10}{2}-\frac{5}{2}$$
$$2.5\stackrel{?}{=}5-2.5$$
$$2.5=2.5$$
True

50. $\begin{cases} 3x-2y=\dfrac{9}{2} \\ \dfrac{x}{2}-\dfrac{3}{4}=2y \end{cases}$

$3x-2y=\dfrac{9}{2}$ The 1st equation
Substitute for $2y$.

$3x-\left(\dfrac{\boldsymbol{x}}{\boldsymbol{2}}-\dfrac{\boldsymbol{3}}{\boldsymbol{4}}\right)=\dfrac{9}{2}$

$3x-\dfrac{x}{2}+\dfrac{3}{4}=\dfrac{9}{2}$

$4(3x)-4\left(\dfrac{x}{2}\right)+4\left(\dfrac{3}{4}\right)=4\left(\dfrac{9}{2}\right)$

$12x-2x+3=18$

$10x+3=18$

$10x+3-\boldsymbol{3}=18-\boldsymbol{3}$

$10x=15$

$\dfrac{10x}{\boldsymbol{10}}=\dfrac{15}{\boldsymbol{10}}$

$x=\dfrac{3}{2}$

$3x-2y=\dfrac{9}{2}$ The 1st equation

$3\left(\dfrac{\boldsymbol{3}}{\boldsymbol{2}}\right)-2y=\dfrac{9}{2}$

$\dfrac{9}{2}-2y=\dfrac{9}{2}$

$\dfrac{9}{2}-2y-\dfrac{9}{2}=\dfrac{9}{2}-\dfrac{9}{2}$

$-2y=0$

$\dfrac{-2y}{-2}=\dfrac{0}{-2}$

$y=0$

The solution is $\left(\dfrac{3}{2},0\right)$.

51. $\begin{cases} y+x=2x+2 \\ 6x-4y=21-y \end{cases}$

Write both equations in standard form.

$\begin{cases} y-x=2 \\ 6x-3y=21 \end{cases}$

Solve for y in the 1st equation.

$\begin{cases} y=2+x \\ 6x-3y=21 \end{cases}$

$6x-3y=21$ The 2nd equation
Substitute for y.

$6x-3(\boldsymbol{2}+\boldsymbol{x})=21$

$6x-6-3x=21$

$3x-6=21$

$3x-6+\boldsymbol{6}=21+\boldsymbol{6}$

$3x=27$

$\dfrac{3x}{\boldsymbol{3}}=\dfrac{27}{\boldsymbol{3}}$

$x=9$

$y=2+x$ The 1st equation

$y=2+\boldsymbol{9}$

$y=11$

The solution is (9, 11).

Check:

$y+x=2x+2$

$11+9\overset{?}{=}2(9)+2$

$20\overset{?}{=}19+2$

$20=20$

True

Check:

$6x-4y=21-y$

$6(9)-4(11)\overset{?}{=}21-11$

$54-44\overset{?}{=}10$

$10=10$

True

52. $\begin{cases} y - x = 3x \\ 2x + 2y = 14 - y \end{cases}$

Write the 2nd equation in standard form.

$\begin{cases} y - x = 3x \\ 2x + 3y = 14 \end{cases}$

Solve for y in the 1st equation.

$\begin{cases} y = 4x \\ 2x + 3y = 14 \end{cases}$

$2x + 3y = 14$ The 2nd equation
Substitute for $2x$.

$2x + 3(\mathbf{4}x) = 14$
$2x + 12x = 14$
$14y = 14$
$\frac{14x}{\mathbf{14}} = \frac{14}{\mathbf{14}}$
$x = 1$

$y = 4x$ The 1st equation
$y = 4(\mathbf{1})$
$y = 4$
The solution is (1, 4).

53. $\begin{cases} 4x + 5y + 1 = -12 + 2x \\ x - 3y + 2 = -3 - x \end{cases}$

Write both equations in standard form.

$\begin{cases} 2x + 5y = -13 \\ 2x - 3y = -5 \end{cases}$

Solve for $2x$ in the 1st equation.

$\begin{cases} 2x = -13 - 5y \\ 2x - 3y = -5 \end{cases}$

$2x - 3y = -5$ The 2nd equation
Substitute for $2x$.

$(-\mathbf{13} - \mathbf{5}y) - 3y = -5$
$-8y - 13 = -5$
$-8y - 13 + \mathbf{13} = -5 + \mathbf{13}$
$-8y = 8$
$\frac{-8y}{\mathbf{-8}} = \frac{8}{\mathbf{-8}}$
$y = -1$

$2x = -13 - 5y$ The 1st equation
$2x = -13 - 5(-\mathbf{1})$
$2x = -13 + 5$
$2x = -8$
$\frac{2x}{2} = \frac{-8}{2}$
$x = -4$

The solution is $(-4, -1)$.

Check:
$4x + 5y + 1 = -12 + 2x$
$4(-4) + 5(-1) + 1 \stackrel{?}{=} -12 + 2(-4)$
$-16 - 5 + 1 \stackrel{?}{=} -12 - 8$
$-20 = -20$
True

Check:
$x - 3y + 2 = -3 - x$
$-4 - 3(-1) + 2 \stackrel{?}{=} -3 - (-4)$
$-4 + 3 + 2 \stackrel{?}{=} -3 + 4$
$1 = 1$
True

54. $\begin{cases} 6x+y=-8+3x-y \\ 3x-y=2y+x-1 \end{cases}$

Write both equations in standard form.

$\begin{cases} 3x+2y=-8 \\ 2x-3y=-1 \end{cases}$

Solve for x in the 1st equation.

$$3x+2y=-8$$
$$3x+2y-\mathbf{2y}=-8-\mathbf{2y}$$
$$3x=-8-2y$$
$$\frac{3x}{\mathbf{3}}=\frac{-8}{\mathbf{3}}-\frac{2y}{\mathbf{3}}$$
$$x=-\frac{8}{3}-\frac{2y}{3}$$

$\begin{cases} x=-\frac{8}{3}-\frac{2y}{3} \\ 2x-3y=-1 \end{cases}$

$2x-3y=-1$ The 2nd equation Substitute for x.

$$2\left(-\frac{\mathbf{8}}{\mathbf{3}}-\frac{\mathbf{2y}}{\mathbf{3}}\right)-3y=-1$$
$$3\left(-\frac{16}{3}\right)-3\left(-\frac{4y}{3}\right)-3y=3(-1)$$
$$-16-4y-9y=-3$$
$$-16-13y=-3$$
$$-16-13y+\mathbf{16}=-3+\mathbf{16}$$
$$-13y=13$$
$$\frac{-13y}{\mathbf{-13}}=\frac{13}{\mathbf{-13}}$$
$$y=-1$$

$3x+2y=-8$ The 1st equation

$$3x+2(\mathbf{-1})=-8$$
$$3x-2+2=-8+2$$
$$3x=-6$$
$$\frac{3x}{3}=\frac{-6}{3}$$
$$x=-2$$

The solution is $(-2,-1)$.

55. $\begin{cases} 3(x-1)+3=8+2y \\ 2(x+1)=8+y \end{cases}$

Distribute in both equations.

$\begin{cases} 3x-3+3=8+2y \\ 2x+2=8+y \end{cases}$

Write the 1st equation in standard form.

Solve for y in the 2nd equation.

$\begin{cases} 3x-2y=8 \\ 2x-6=y \end{cases}$

$3x-2y=8$ The 1st equation Substitute for y.

$$3x-2(\mathbf{2x-6})=8$$
$$3x-4x+12=8$$
$$-x+12=8$$
$$-x+12-\mathbf{12}=8-\mathbf{12}$$
$$-x=-4$$
$$\frac{-x}{\mathbf{-1}}=\frac{-4}{\mathbf{-1}}$$
$$x=4$$

$2x-6=y$ The 2nd equation

$$2(\mathbf{4})-6=y$$
$$8-6=y$$
$$2=y$$

The solution is (4, 2).

Check:

$3(x-1)+3=8+2y$

$3(4-1)+3\overset{?}{=}8+2(2)$

$3(3)+3\overset{?}{=}8+4$

$9+3\overset{?}{=}12$

$12=12$

True

Check:

$2(x+1)=8+y$

$2(4+1)\overset{?}{=}8+2$

$2(5)\overset{?}{=}10$

$10=10$

True

56. $\begin{cases} 4(x-2)=19-5y \\ 3(x-2)-2y=-y \end{cases}$

Distribute in both equations.

$\begin{cases} 4x-8=19-5y \\ 3x-6-2y=-y \end{cases}$

Write the 1st equation in standard form.

Solve for y in the 2nd equation.

$\begin{cases} 4x+5y=27 \\ 3x-6=y \end{cases}$

$4x+5y=27$ The 1st equation Substitute for y.

$4x+5(\mathbf{3x}-\mathbf{6})=27$

$4x+15x-30=27$

$19x-30=27$

$19x-30+\mathbf{30}=27+\mathbf{30}$

$19x=57$

$\frac{19x}{\mathbf{19}}=\frac{57}{\mathbf{19}}$

$x=3$

$3x-6=y$ The 2nd equation

$3(\mathbf{3})-6=y$

$9-6=y$

$3=y$

The solution is (3, 3).

Solve each system by substituion.

57. $\begin{cases} 2a+4b=-24 \\ a=20-2b \end{cases}$

$2a+4b=-24$ The 1st equation Substitute for a.

$2(\mathbf{20}-\mathbf{2b})+4b=-24$

$40-4b+4b=-24$ The variables drop out.

$40=-24$

False

No Solution.

58. $\begin{cases} 3a+6b=-15 \\ a=-2b-5 \end{cases}$

$3a+6b=-15$ The 1st equation Substitute for a.

$3(\mathbf{-2b}-\mathbf{5})+6b=-15$

$-6b-15+6b=-15$ The variables drop out.

$-15=-15$

True

The system has infinitely many solutions.

59. $\begin{cases} y-3x=-5 \\ 21x=7y+35 \end{cases}$

Solve for y in the 1st equation.

$\begin{cases} y=3x-5 \\ 21x=7y+35 \end{cases}$

$21x=7y+35$ The 2nd equation Substitute for y.

$21x=7(\mathbf{3x}-\mathbf{5})+35$

$21x=21x-35+35$

$21x=21x$

$21x-\mathbf{21x}=21x-\mathbf{21x}$ The variables drop out.

$0=0$

True

The system has infinitely many solutions.

60. $\begin{cases} 8y=15-4x \\ x+2y=4 \end{cases}$

Solve for x in the 2nd equation.

$\begin{cases} 8y=15-4x \\ x=4-2y \end{cases}$

$8y=15-4x$ The 1st equation Substitute for x.

$8y=15-4(\mathbf{4}-\mathbf{2y})$

$8y=15-16+8y$

$8y=-1+8y$

$8y-\mathbf{8y}=-1+8y-\mathbf{8y}$ The variables drop out.

$0=-1$

False

No Solution.

61. $\begin{cases} 6-y=4x \\ 2y=-8x-20 \end{cases}$

Solve for y in the 1st equation.

$\begin{cases} 6-4x=y \\ 2y=-8x-20 \end{cases}$

$2y=-8x-20$ The 2nd equation

Substitute for y.

$2(6-4x)=-8x-20$

$12-8x=-8x-20$

$12-8x+\mathbf{8x}=-8x-20+\mathbf{8x}$ The variables drop out.

$12=-20$

False

No Solution.

62. $\begin{cases} 9x=3y+12 \\ 4=3x-y \end{cases}$

Solve for y in the 2nd equation.

$\begin{cases} 9x=3y+12 \\ y=3x-4 \end{cases}$

$9x=3y+12$ The 1st equation

Substitute for y.

$9x=3(\mathbf{3x}-\mathbf{4})+12$

$9x=9x-12+12$

$9x=9x$

$9x-\mathbf{9x}=9x-\mathbf{9x}$ The variables drop out.

$0=0$

True

The system has infinitely many solutions.

63. $\begin{cases} x=-3y+6 \\ 2x+4y=6+x+y \end{cases}$

Write the 2nd equation in standard form.

$\begin{cases} x=-3y+6 \\ x+3y=6 \end{cases}$

$x+3y=6$ The 2nd equation

Substitute for x.

$(-\mathbf{3y}+\mathbf{6})+3y=6$ The variables drop out.

$6=6$

True

The system has infinitely many solutions.

64. $\begin{cases} 2x-y=x+y \\ -2x+4y=6 \end{cases}$

Solve for x in the 1st equation.

$\begin{cases} x=2y \\ -2x+4y=6 \end{cases}$

$-2x+4y=6$ The 2nd equation

Substitute for x.

$-2(\mathbf{2y})+4y=6$

$-4y+4y=6$ The variables drop out.

$0=6$

False

No Solution.

TRY IT YOURSELF

Solve each system, if possible. If a system has no solution or infinitely many solutions, so state.

65. $\begin{cases} -y=11-3x \\ 2x+5y=-4 \end{cases}$

Solve for y in the 1st equation.

$\begin{cases} y=3x-11 \\ 2x+5y=-4 \end{cases}$

$2x+5y=-4$ The 2nd equation

Substitute for y.

$2x+5(\mathbf{3x}-\mathbf{11})=-4$

$2x+15x-55=-4$

$17x-55=-4$

$17x-55+\mathbf{55}=-4+\mathbf{55}$

$17x=51$

$\frac{17x}{\mathbf{17}}=\frac{51}{\mathbf{17}}$

$x=3$

$y=3x-11$ The 1st equation

$y=3(\mathbf{3})-11$

$y=9-11$

$y=-2$

The solution is $(3,-2)$.

Check:

$-y=11-3x$

$-(-2)\overset{?}{=}11-3(3)$

$2\overset{?}{=}11-9$

$2=2$

True

Check:

$2x+5y=-4$

$2(3)+5(-2)\overset{?}{=}-4$

$6-10\overset{?}{=}-4$

$-4=-4$

True

66. $\begin{cases} -x = 10 - 3y \\ 2x + 8y = -6 \end{cases}$

Solve for x in the 1st equation.

$\begin{cases} x = 3y - 10 \\ 2x + 8y = -6 \end{cases}$

$2x + 8y = -6$ The 2nd equation
Substitute for x.

$2(\mathbf{3}y - \mathbf{10}) + 8y = -6$
$6y - 20 + 8y = -6$
$14y - 20 = -6$
$14y - 20 + \mathbf{20} = -6 + \mathbf{20}$
$14y = 14$
$\frac{14y}{\mathbf{14}} = \frac{14}{\mathbf{14}}$
$y = 1$

$x = 3y - 10$ The 1st equation
$x = 3(\mathbf{1}) - 10$
$x = 3 - 10$
$x = -7$

The solution is $(-7, 1)$.

67. $\begin{cases} \frac{x}{2} + \frac{y}{6} = \frac{2}{3} \\ \frac{x}{3} - \frac{y}{4} = \frac{1}{12} \end{cases}$

Clear both equations of fractions.

$\begin{cases} 6\left(\frac{x}{2}\right) + 6\left(\frac{y}{6}\right) = 6\left(\frac{2}{3}\right) \\ 12\left(\frac{x}{3}\right) - 12\left(\frac{y}{4}\right) = 12\left(\frac{1}{12}\right) \end{cases}$

$\begin{cases} 3x + y = 4 \\ 4x - 3y = 1 \end{cases}$

Solve for y in the 1st equation.

$\begin{cases} y = 4 - 3x \\ 4x - 3y = 1 \end{cases}$

$4x - 3y = 1$ The 2nd equation
Substitute for y.

$4x - 3(\mathbf{4} - \mathbf{3}x) = 1$
$4x - 12 + 9x = 1$
$13x - 12 = 1$
$13x - 12 + \mathbf{12} = 1 + \mathbf{12}$
$13x = 13$
$\frac{13x}{\mathbf{13}} = \frac{13}{\mathbf{13}}$
$x = 1$

$y = 4 - 3x$ The 1st equation
$y = 4 - 3(\mathbf{1})$
$y = 4 - 3$
$y = 1$

The solution is $(1,1)$.

Check:

$\frac{x}{2} + \frac{y}{6} = \frac{2}{3}$
$\frac{1}{2} + \frac{1}{6} \stackrel{?}{=} \frac{2}{3}$
$\frac{3}{6} + \frac{1}{6} \stackrel{?}{=} \frac{2}{3}$
$\frac{4}{6} \stackrel{?}{=} \frac{2}{3}$
$\frac{2}{3} = \frac{2}{3}$
True

Check:

$\frac{x}{3} - \frac{y}{4} = \frac{1}{12}$
$\frac{1}{3} - \frac{1}{4} \stackrel{?}{=} \frac{1}{12}$
$\frac{4}{12} - \frac{3}{12} \stackrel{?}{=} \frac{1}{12}$
$\frac{1}{12} = \frac{1}{12}$
True

68. $\begin{cases} \frac{c}{2}+\frac{d}{14}=1 \\ \frac{c}{5}-\frac{d}{2}=-\frac{33}{10} \end{cases}$

Clear both equations of fractions.

$$\begin{cases} 14\left(\frac{c}{2}\right)+14\left(\frac{d}{14}\right)=14(1) \\ 10\left(\frac{c}{5}\right)-10\left(\frac{d}{2}\right)=10\left(-\frac{33}{10}\right) \end{cases}$$

$$\begin{cases} 7c+d=14 \\ 2c-5d=-33 \end{cases}$$

Solve for d in the 1st equation.

$$\begin{cases} d=14-7c \\ 2c-5d=-33 \end{cases}$$

$2c-5d=-33$ The 2nd equation Substitute for d.

$2c-5(\mathbf{14}-\mathbf{7}\boldsymbol{c})=-33$

$2c-70+35c=-33$

$37c-70=-33$

$37c-70+\mathbf{70}=-33+\mathbf{70}$

$37c=37$

$\frac{37c}{\mathbf{37}}=\frac{37}{\mathbf{37}}$

$c=1$

$d=14-7c$ The 1st equation

$d=14-7(\mathbf{1})$

$d=14-7$

$d=7$

The solution is (1,7).

69. $\begin{cases} y-4=2x \\ y=2x+2 \end{cases}$

$y-4=2x$ The 1st equation Substitute for y.

$(\mathbf{2}\boldsymbol{x}+\mathbf{2})-4=2x$

$2x-2=2x$

$2x-2-\mathbf{2}\boldsymbol{x}=2x-\mathbf{2}\boldsymbol{x}$ The variables drop out.

$-2=0$

False

No Solution.

70. $\begin{cases} x+3y=6 \\ x=-3y+6 \end{cases}$

$x+3y=6$ The 1st equation Substitute for x.

$(-\mathbf{3}\boldsymbol{y}+\mathbf{6})+3y=6$ The variables drop out.

$6=6$

True

The system has infinitely many solutions.

71. $\begin{cases} a+b=1 \\ a-2b=-1 \end{cases}$

Solve for a in the 1st equation.

$$\begin{cases} a=1-b \\ a-2b=-1 \end{cases}$$

$a-2b=-1$ The 2nd equation Substitute for a.

$(\mathbf{1}-\boldsymbol{b})-2b=-1$

$1-3b=-1$

$1-3b-\mathbf{1}=-1-\mathbf{1}$

$-3b=-2$

$\frac{-3b}{-\mathbf{3}}=\frac{-2}{-\mathbf{3}}$

$b=\frac{2}{3}$

$a=1-b$ The 1st equation

$a=1-\frac{\mathbf{2}}{\mathbf{3}}$

$a=\frac{3}{3}-\frac{2}{3}$

$a=\frac{1}{3}$

The solution is $\left(\frac{1}{3},\frac{2}{3}\right)$.

Check:

$a+b=1$

$\frac{1}{3}+\frac{2}{3}\stackrel{?}{=}1$

$\frac{3}{3}\stackrel{?}{=}1$

$1=1$

True

Check:

$a-2b=-1$

$\frac{1}{3}-2\left(\frac{2}{3}\right)\stackrel{?}{=}-1$

$\frac{1}{3}-\frac{4}{3}\stackrel{?}{=}-1$

$-\frac{3}{3}\stackrel{?}{=}-1$

$-1=-1$

True

72. $\begin{cases} 2b-a=-1 \\ 3a+10b=-1 \end{cases}$

Solve for a in the 1st equation.

$\begin{cases} 2b+1=a \\ 3a+10b=-1 \end{cases}$

$3a+10b=-1$ The 2nd equation

Substitute for a.

$3(\mathbf{2b+1})+10b=-1$

$6b+3+10b=-1$

$16b+3=-1$

$16b+3-\mathbf{3}=-1-\mathbf{3}$

$16b=-4$

$\frac{16b}{\mathbf{16}}=\frac{-4}{\mathbf{16}}$

$b=-\frac{1}{4}$

$2b+1=a$ The 1st equation

$2\left(-\frac{\mathbf{1}}{\mathbf{4}}\right)+1=a$

$-\frac{1}{2}+1=a$

$-\frac{1}{2}+\frac{2}{2}=a$

$\frac{1}{2}=a$

The solution is $\left(\frac{1}{2},-\frac{1}{4}\right)$.

73. $\begin{cases} x=7y-10 \\ 2x-14y+20=0 \end{cases}$

Put the 2nd equation in standard form.

$\begin{cases} x=7y-10 \\ 2x-14y=-20 \end{cases}$

$2x-14y=-20$ The 2nd equation

Substitute for x.

$2(\mathbf{7y-10})-14y=-20$

$14y-20-14y=-20$ The variables drop out.

$-20=-20$

True

The system has infinitely many solutions.

74. $\begin{cases} y-1=5x \\ 10x-2y=2 \end{cases}$

Solve for y in the 1st equation.

$\begin{cases} y=5x+1 \\ 10x-2y=2 \end{cases}$

$10x-2y=2$ The 2nd equation

Substitute for y.

$10x-2(\mathbf{5x+1})=2$

$10x-10x-2=2$ The variables drop out.

$-2=2$

False

No Solution.

75. $\begin{cases} 4x+1=2x+5+y \\ 2x+2y=5x+y+6 \end{cases}$

Solve for y in the 1st equation.

Write the 2nd equation in standard form.

$\begin{cases} 2x-4=y \\ -3x+y=6 \end{cases}$

$-3x+y=6$ The 2nd equation

Substitute for y.

$-3x+(\mathbf{2x-4})=6$

$-x-4=6$

$-x-4+\mathbf{4}=6+\mathbf{4}$

$-x=10$

$\frac{-x}{\mathbf{-1}}=\frac{10}{\mathbf{-1}}$

$x=-10$

$2x-4=y$ The 1st equation

$2(\mathbf{-10})-4=y$

$-20-4=y$

$-24=y$

The solution is $(-10,-24)$.

Check:

$4x+1=2x+5+y$

$4(-10)+1\overset{?}{=}2(-10)+5+(-24)$

$-40+1\overset{?}{=}-20+5-24$

$-39=-39$

True

Check:

$2x+2y=5x+y+6$

$2(-10)+2(-24)\overset{?}{=}5(-10)+(-24)+6$

$-20-48\overset{?}{=}-50-24+6$

$-68=-68$

True

76. $\begin{cases} 6x = 2(y+20) + 5x \\ 5(x-1) = 3y + 4(x+10) \end{cases}$

Distribute both equations.

$\begin{cases} 6x = 2y + 40 + 5x \\ 5x - 5 = 3y + 4x + 40 \end{cases}$

Solve the 1st equation for x.

Write the 2nd equation in standard form.

$\begin{cases} x = 2y + 40 \\ x - 3y = 45 \end{cases}$

$x - 3y = 45$ The 2nd equation
Substitute for x.

$(\mathbf{2y + 40}) - 3y = 45$
$-y + 40 = 45$
$-y + 40 - \mathbf{40} = 45 - \mathbf{40}$
$-y = 5$
$\frac{-y}{\mathbf{-1}} = \frac{5}{\mathbf{-1}}$
$y = -5$

$x = 2y + 40$ The 1st equation
$x = 2(\mathbf{-5}) + 40$
$x = -10 + 40$
$x = 30$

The solution is $(30, -5)$.

77. $\begin{cases} 2a + 3b = 7 \\ 6a - b = 1 \end{cases}$

Solve for b in the 2nd equation.

$\begin{cases} 2a + 3b = 7 \\ 6a - 1 = b \end{cases}$

$2a + 3b = 7$ The 1st equation
Substitute for b.

$2a + 3(\mathbf{6a - 1}) = 7$
$2a + 18a - 3 = 7$
$20a - 3 = 7$
$20a - 3 + \mathbf{3} = 7 + \mathbf{3}$
$20a = 10$
$\frac{20a}{\mathbf{20}} = \frac{10}{\mathbf{20}}$
$a = \frac{1}{2}$

$6a - 1 = b$ The 2nd equation
$6\left(\frac{\mathbf{1}}{\mathbf{2}}\right) - 1 = b$
$3 - 1 = b$
$2 = b$

The solution is $\left(\frac{1}{2}, 2\right)$.

Check:
$2a + 3b = 7$
$2\left(\frac{1}{2}\right) + 3(2) \stackrel{?}{=} 7$
$1 + 6 \stackrel{?}{=} 7$
$7 = 7$
True

Check:
$6a - b = 1$
$6\left(\frac{1}{2}\right) - 2 \stackrel{?}{=} 1$
$3 - 2 \stackrel{?}{=} 1$
$1 = 1$
True

78. $\begin{cases} 3a + 5b = -6 \\ 5b - a = -3 \end{cases}$

Solve for a in the 2nd equation.

$\begin{cases} 3a + 5b = -6 \\ 5b + 3 = a \end{cases}$

$3a + 5b = -6$ The 1st equation
Substitute for a.

$3(\mathbf{5b + 3}) + 5b = -6$
$15b + 9 + 5b = -6$
$20b + 9 = -6$
$20b + 9 - \mathbf{9} = -6 - \mathbf{9}$
$20b = -15$
$\frac{20b}{\mathbf{20}} = \frac{-15}{\mathbf{20}}$
$b = -\frac{3}{4}$

$5b + 3 = a$ The 2nd equation
$5\left(-\frac{\mathbf{3}}{\mathbf{4}}\right) + 3 = a$
$-\frac{15}{4} + \frac{12}{4} = a$
$-\frac{3}{4} = a$

The solution is $\left(-\frac{3}{4}, -\frac{3}{4}\right)$.

79. $\begin{cases} 2x-3y=-4 \\ x=-\frac{3}{2}y \end{cases}$

$2x-3y=-4$ The 1st equation
Substitute for x.

$2\left(-\frac{\mathbf{3}}{\mathbf{2}}y\right)-3y=-4$

$-3y-3y=-4$

$-6y=-4$

$\frac{-6y}{\mathbf{-6}}=\frac{-4}{\mathbf{-6}}$

$y=\frac{2}{3}$

$x=-\frac{3}{2}y$ The 2nd equation

$x=-\frac{3}{2}\left(\frac{\mathbf{2}}{\mathbf{3}}\right)$

$x=1$

The solution is $\left(-1,\frac{2}{3}\right)$.

Check:

$2x-3y=-4$

$2(-1)-3\left(\frac{2}{3}\right)\stackrel{?}{=}-4$

$-2-2\stackrel{?}{=}-4$

$-4=-4$

True

Check:

$x=-\frac{3}{2}y$

$-1\stackrel{?}{=}-\frac{3}{2}\left(\frac{2}{3}\right)$

$-1=-1$

True

80. $\begin{cases} x=-\frac{3}{8}y \\ 8x-3y=4 \end{cases}$

$8x-3y=4$ The 2nd equation
Substitute for x.

$8\left(-\frac{\mathbf{3}}{\mathbf{8}}y\right)-3y=4$

$-3y-3y=4$

$-6y=4$

$\frac{-6y}{\mathbf{-6}}=\frac{4}{\mathbf{-6}}$

$y=-\frac{2}{3}$

$x=-\frac{3}{8}y$ The 1st equation

$x=-\frac{3}{8}\left(-\frac{\mathbf{2}}{\mathbf{3}}\right)$

$x=\frac{1}{4}$

The solution is $\left(\frac{1}{4},-\frac{2}{3}\right)$.

APPLICATIONS

81. OFFROADING

Let a = angle of approach in degrees
d = angle of departure in degrees

$\begin{cases} a+d=77 \\ a=d+3 \end{cases}$

$a+d=77$ The 1st equation
Substitute for a.

$\boldsymbol{d}+\mathbf{3}+d=77$

$2d+3=77$

$2d+3-\mathbf{3}=77-\mathbf{3}$

$2d=74$

$\frac{2d}{\mathbf{2}}=\frac{74}{\mathbf{2}}$

$d=37$

$a=d+3$ The 2nd equation

$a=\mathbf{37}+3$

$a=40$

The angle of approach is 40°.

The angle of departure is 37°.

82. HIGH SCHOOL SPORTS

Let x = number of years after 2000
y = number of participants

$$\begin{cases} y = 5,071x + 329,100 \\ y = 7,786x + 277,515 \end{cases}$$

$y = 5,071x + 329,100$ The 1st equation
Substitute for y.

$7,786x + 277,515 - \mathbf{277,515} = 5,071x + 329,100 - \mathbf{277,515}$

$$7,786x = 5,071x + 51,585$$
$$7,786x - \mathbf{5,071x} = 5,071x + 51,585 - \mathbf{5,071x}$$
$$2,715x = 51,585$$
$$\frac{2,715x}{\mathbf{2,175}} = \frac{51,585}{\mathbf{2,175}}$$
$$x = 19$$

The year would be 2019 when the number of boys and girls participating in soccer would be the same.

WRITING

83. Answers will vary.

84. Answers will vary.

85. Answers will vary.

86. Answers will vary.

REVIEW

87. Find the prime factors of 189.

$$189 = 3 \cdot 63$$
$$= 3 \cdot 7 \cdot 9$$
$$= 3 \cdot 7 \cdot 3 \cdot 3$$
$$= 3^3 \cdot 7$$

88. Complete each statement. For any nonzero number a,

a. $\frac{a}{a} = \mathbf{1}$ b. $\frac{a}{1} = \boldsymbol{a}$

c. $\frac{0}{a} = \mathbf{0}$ d. $\frac{a}{0} = \mathbf{undefined}$

89. Simplify:

$$\frac{30}{36} = \frac{2 \cdot 3 \cdot 5}{2 \cdot 2 \cdot 3 \cdot 3}$$
$$= \frac{\overset{1}{\cancel{2}} \cdot \overset{1}{\cancel{3}} \cdot 5}{\underset{1}{\cancel{2}} \cdot 2 \cdot \underset{1}{\cancel{3}} \cdot 3}$$
$$= \frac{5}{6}$$

90. Add:

$$\frac{5}{12} + \frac{1}{4} = \frac{5}{12} + \frac{1}{4}\left(\frac{\mathbf{3}}{\mathbf{3}}\right)$$
$$= \frac{5+3}{12}$$
$$= \frac{8}{12}$$
$$= \frac{\overset{1}{\cancel{2}} \cdot \overset{1}{\cancel{2}} \cdot 2}{\underset{1}{\cancel{2}} \cdot \underset{1}{\cancel{2}} \cdot 3}$$
$$= \frac{2}{3}$$

91. Multiply:

$$\frac{7}{8} \cdot \frac{3}{5} = \frac{7 \cdot 3}{8 \cdot 5}$$
$$= \frac{21}{40}$$

92. Divide:

$$\frac{1}{3} \div \frac{4}{5} = \frac{1}{3} \cdot \frac{5}{4}$$
$$= \frac{1 \cdot 5}{3 \cdot 4}$$
$$= \frac{5}{12}$$

CHALLENGE PROBLEMS

93. $\begin{cases} \dfrac{6x-1}{3}-\dfrac{5}{3}=\dfrac{3y+1}{2} \\ \dfrac{1+5y}{4}+\dfrac{x+3}{4}=\dfrac{17}{2} \end{cases}$

Clear both equations of fractions.

$$\begin{cases} 6\left(\dfrac{6x-1}{3}\right)-6\left(\dfrac{5}{3}\right)=6\left(\dfrac{3y+1}{2}\right) \\ 4\left(\dfrac{1+5y}{4}\right)+4\left(\dfrac{x+3}{4}\right)=4\left(\dfrac{17}{2}\right) \end{cases}$$

$$\begin{cases} 12x-2-10=9y+3 \\ 1+5y+x+3=34 \end{cases}$$

Write the 1st equation in standard form.

Solve for x in the 2nd equation.

$$\begin{cases} 12x-9y=15 \\ x=30-5y \end{cases}$$

$12x-9y=15$ The 1st equation
Substitute for x.

$$\begin{aligned} 12(\mathbf{30}-\mathbf{5}\boldsymbol{y})-9y&=15 \\ 360-60y-9y&=15 \\ 360-69y&=15 \\ 360-69y-\mathbf{360}&=15-\mathbf{360} \\ -69y&=-345 \\ \frac{-69y}{\mathbf{-69}}&=\frac{-345}{\mathbf{-69}} \\ y&=5 \end{aligned}$$

$x=30-5y$ The 2nd equation

$$\begin{aligned} x&=30-5(\mathbf{5}) \\ x&=30-25 \\ x&=5 \end{aligned}$$

The solution is (5, 5).

The solution checks.

94. $\begin{cases} 0.5x+0.5y=6 \\ 0.001x-0.001y=-0.004 \end{cases}$

Multiply the 1st equation by 10.

Multiply the 2nd equation by 1,000.

$$\begin{cases} 5x+5y=60 \\ x-y=-4 \end{cases}$$

Solve the 2nd equation for x.

$$\begin{cases} 5x+5y=60 \\ x=y-4 \end{cases}$$

$5x+5y=60$ The 1st equation
Substitute for x.

$$\begin{aligned} 5(\boldsymbol{y}-\mathbf{4})+5y&=60 \\ 5y-20+5y&=60 \\ 10y-20&=60 \\ 10y-20+20&=60+20 \\ 10y&=80 \\ \frac{10y}{10}&=\frac{80}{10} \\ y&=8 \end{aligned}$$

$x=y-4$ The 2nd equation

$$\begin{aligned} x&=\mathbf{8}-4 \\ x&=4 \end{aligned}$$

The solution is (4, 8).

The solution checks.

95. $\begin{cases} \frac{1}{2}x = y + 3 \\ x - 2y = 6 \end{cases}$

Step 1: Solve the 2nd equation for *x*.

$x = 2y + 6$

To find one solution, let $y = -3$.

$x = 2y + 6$

$x = 2(-3) + 6$

$x = -6 + 6$

$x = 0$

The 1st solution is $(0, -3)$.

Step 2: Solve the 2nd equation for *x*.

$x = 2y + 6$

To find one solution let $y = 0$.

$x = 2y + 6$

$x = 2(0) + 6$

$x = 0 + 6$

$x = 6$

The 2nd solution is (6, 0).

Step 3: Solve the 2nd equation for *x*.

$x = 2y + 6$

To find one solution let $y = -2$.

$x = 2y + 6$

$x = 2(-2) + 6$

$x = -4 + 6$

$x = 2$

The 3rd solution is $(2, -2)$.

You can check each of these solutions by substituting them into the 1st equation.

96. $\begin{cases} y = -2 \\ x = 5 \end{cases}$

No, because there is no way to make a substitution.

Each equation gives each part of the ordered pair, $(5, -2)$. Or it can be solved by graphing each equation.