

SECTION 4.4 STUDY SET

VOCABULARY

Fill in the blanks.

- Two angles are said to be complementary if the sum of their measures is 90° . Two angles are said to be supplementary if the sum of their measures is 180° .
- Problems that involve moving vehicles are called uniform motion problems. Problems that involve combining ingredients are called mixture problems. Problems that involve collections of different items having different values are called number- value problems.

CONCEPTS

- A length of pipe is to be cut into two pieces. The longer piece is to be 1 foot less than twice the shorter piece. Write two equations that model the situation. $x + y = 20$, $y = 2x - 1$
- Two angles are complementary. The measure of the larger angle is four times the measure of the smaller angle. Write two equations that model the situation. $x + y = 90$, $x = 4y$
- Two angles are supplementary. The measure of the smaller angle is 25° less than the measure of the larger angle. Write two equations that model the situation. $x + y = 180$, $y = x - 25$
- The perimeter of a ping pong table is 28 feet. The length is 4 feet more than the width. Write two equations that model the situation.
 $2l + 2w = 28$, $l = w + 4$
- Let x = the cost of a chicken taco and y = the cost of a beef taco. Write an equation that models the offer shown in the advertisement.
 $5x + 2y = 10$

8. a.

Principal \square Rate \square Time = Interest

City Bank	x	5%	1 yr	0.05x
USA Savings	y	11%	1 yr	0.11y

- A total of \$50,000 was deposited in the two accounts. Use that information to write an equation about the principal.

$$x + y = 50,000$$

- A total of \$4,300 was earned by the two accounts. Use that information to write an equation about the interest.

$$0.05x + 0.11y = 4,300$$

- For each case below, write an algebraic expression that represents the speed of the canoe in miles per hour if its speed in still water is x mph. $x + c$, $x - c$
- Complete the table.

Rate \square Time = Distance

With wind	$x + y$	3	3(x + y)
Against wind	$x - y$	5	5(x - y)

- If the contents of the two test tubes are poured into a third tube, how much solution will the third tube contain? (mL stands for milliliter. A milliliter is about two drops from an eyedropper.) $(x + y)$ mL
- Which of the following strengths could the mixture possibly be: 27%, 33%, or 44% acid solution? **33%**

12. a. Complete the table.

Amt \square Strength = Amt of salt

Weak	x	0.01	0.01x
Strong	y	0.06	0.06y
Mix	12	0.03	12(0.03)

- Use the information from the Amount column to write an equation. $x + y = 12$
- Use the information from the Amount of salt column to write an equation.

$$0.01x + 0.06y = 12(0.03)$$

GUIDED PRACTICE

13. COMPLEMENTARY ANGLES

Analyze the Problem

- Two angles are complementary (90°).
- One angle is 10° more than three times the measure of the other.
- Find the measure of each angle.

Form the System

Let x = the measure of one \angle
 y = the measure of the other \angle

The measure of one angle plus the measure of the other angle is complementary.

$$x + y = 90$$

One angle is 10° more than 3 times the other angle.

$$x = 10 + 3y$$

Solve the System

$$\begin{cases} x + y = 90 \\ x = 10 + 3y \end{cases}$$

Write the 2nd equation in standard form.

$$\begin{cases} x + y = 90 \\ x - 3y = 10 \end{cases}$$

Eliminate x .

$$\begin{array}{r} x + y = 90 \\ -x + 3y = -10 \\ \hline 4y = 80 \\ \frac{4y}{4} = \frac{80}{4} \\ y = 20 \end{array}$$

$$x = 10 + 3y \text{ The 2}^{\text{nd}} \text{ equation}$$

$$x = 10 + 3(20)$$

$$x = 10 + 60$$

$$x = 70$$

State the Conclusion

The measure of one angle is 70° .

The measure of the other angle is 20° .

Check the Results

$$\begin{array}{l|l} 70 + 20 = 90 & 70 = 10 + 3(20) \\ & 70 = 70 \end{array}$$

The results check.

14. SUPPLEMENTARY ANGLES

Analyze the Problem

- Two angles are supplementary (180°).
- One angle is 20° less than 19 times the measure of the other.
- Find the measure of each angle.

Form the System

Let x = the measure of one \angle
 y = the measure of the other \angle

The measure of one angle plus the measure of the other angle is supplementary.

$$x + y = 180$$

One angle is 20° less than 19 times the other angle.

$$x = 19y - 20$$

Solve the System

$$\begin{cases} x + y = 180 \\ x = 19y - 20 \end{cases}$$

Write the 2nd equation in standard form.

$$\begin{cases} x + y = 90 \\ x - 19y = -20 \end{cases}$$

Eliminate x .

$$\begin{array}{r} x + y = 180 \\ -x + 19y = -20 \\ \hline 20y = 200 \\ \frac{20y}{20} = \frac{200}{20} \\ y = 10 \end{array}$$

$$x = 19y - 20 \text{ The 2}^{\text{nd}} \text{ equation}$$

$$x = 19(10) - 20$$

$$x = 190 - 20$$

$$x = 170$$

State the Conclusion

The measure of one angle is 170° .

The measure of the other angle is 10° .

Check the Results

The results check.

15. SUPPLEMENTARY ANGLES

Analyze the Problem

- Two angles are supplementary (180°).
- The difference of the measures of two supplementary angles is 80° .
- Find the measure of each angle.

Form the System

Let x = the measure of one \angle (larger \angle)
 y = the measure of the other \angle (smaller \angle)

$$\begin{array}{|c|} \hline \text{The measure} \\ \text{of one angle} \\ \hline \end{array} \text{ plus } \begin{array}{|c|} \hline \text{the measure of} \\ \text{the other angle} \\ \hline \end{array} \text{ is } \begin{array}{|c|} \hline \text{supplementary.} \\ \hline \end{array}$$

$$x + y = 180$$

$$\begin{array}{|c|} \hline \text{The difference of the} \\ \text{measures of the 2 angles} \\ \hline \end{array} \text{ is } \begin{array}{|c|} \hline 80^\circ \\ \hline \end{array}$$

$$x - y = 80$$

Solve the System

$$\begin{cases} x + y = 180 \\ x - y = 80 \end{cases}$$

Eliminate y .

$$x + y = 180$$

$$x - y = 80$$

$$2x = 260$$

$$\frac{2x}{2} = \frac{260}{2}$$

$$x = 130$$

$$x + y = 180 \text{ The 1}^{\text{st}} \text{ equation}$$

$$130 + y = 180$$

$$130 + y - 130 = 180 - 130$$

$$y = 50$$

State the Conclusion

The measure of one angle is 130° .

The measure of the other angle is 50° .

Check the Results

$$\begin{array}{l|l} 130 + 50 = 180 & 130 - 50 = 80 \\ 180 = 180 & 80 = 80 \end{array}$$

The results check.

16. COMPLEMENTARY ANGLES

Analyze the Problem

- Two angles are complementary (90°).
- One angle is 15° more than one-half of the measure of the other.
- Find the measure of each angle.

Form the System

Let x = the measure of one \angle
 y = the measure of the other \angle

$$\begin{array}{|c|} \hline \text{The measure} \\ \text{of one angle} \\ \hline \end{array} \text{ plus } \begin{array}{|c|} \hline \text{the measure of} \\ \text{the other angle} \\ \hline \end{array} \text{ is } \begin{array}{|c|} \hline \text{complementary.} \\ \hline \end{array}$$

$$x + y = 90$$

$$\begin{array}{|c|} \hline \text{One} \\ \text{angle} \\ \hline \end{array} \text{ is } \begin{array}{|c|} \hline 15^\circ \\ \hline \end{array} \text{ more than } \begin{array}{|c|} \hline \text{one-half of the} \\ \text{other angle.} \\ \hline \end{array}$$

$$x = 15 + \frac{1}{2}y$$

Solve the System

$$\begin{cases} x + y = 90 \\ x = 15 + \frac{1}{2}y \end{cases}$$

Multiply both sides of the 2nd equation by 2.

Write the 2nd equation in standard form.

$$\begin{cases} x + y = 90 \\ 2x - y = 30 \end{cases}$$

Eliminate y .

$$x + y = 90$$

$$2x - y = 30$$

$$3x = 120$$

$$\frac{3x}{3} = \frac{120}{3}$$

$$x = 40$$

$$x + y = 90 \text{ The 1}^{\text{st}} \text{ equation}$$

$$40 + y = 90$$

$$40 + y - 40 = 90 - 40$$

$$y = 50$$

State the Conclusion

The measure of one angle is 40° .

The measure of the other angle is 50° .

Check the Results

The results check.

APPLICATION

Write a system of two equations in two variables to solve each problem.

17. TREE TRIMMING

Analyze the Problem

- The total length of both arms is 51 ft.
- The upper arm is 7 ft. shorter than the lower arm.
- Find the length of each arm.

Form the System

Let x = length of upper arm in feet

(shorter length)

y = length of lower arm in feet

(longer length)

The length of the upper arm	plus	the length of the lower arm	is	51 feet.
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x	+	y	=	51
The upper arm	is	7 feet	shorter than	the lower arm.
x	=	y	-	7

Solve the System

$$\begin{cases} x + y = 51 \\ x = y - 7 \end{cases}$$

Write the 2nd equation in standard form.

$$\begin{cases} x + y = 51 \\ x - y = -7 \end{cases}$$

Eliminate y .

$$\begin{array}{r} x + y = 51 \\ x - y = -7 \\ \hline 2x = 44 \\ \frac{2x}{2} = \frac{44}{2} \\ x = 22 \end{array}$$

$$x + y = 51 \text{ The 1}^{\text{st}} \text{ equation}$$

$$22 + y = 51$$

$$22 + y - 22 = 51 - 22$$

$$y = 29$$

State the Conclusion

The measure of the upper arm is 22 ft.

The measure of the lower arm is 29 ft.

Check the Results

$$\begin{array}{l|l} 22 + 29 = 51 & 22 = 29 - 7 \\ 51 = 51 & 22 = 22 \end{array}$$

The results check.

18. ALASKA

Analyze the Problem

- The total lengths both parts is 1,422 mi.
- The Canadian part minus the Alaskan part is 1,020 mi.
- Find the length of each arm.

Form the System

Let x = length of Alaskan part of highway in miles (shorter length)

y = length of Canadian part of highway in miles (longer length)

The length of the Alaskan part	plus	the length of the Canadian part	is	1,422 miles.
x	+	y	=	1,422

The Canadian part	minus	the Alaskan part	is	1,020 miles.
y	-	x	=	1,020

Solve the System

$$\begin{cases} x + y = 1,422 \\ y - x = 1,020 \end{cases}$$

Write the 2nd equation in standard form.

$$\begin{cases} x + y = 1,422 \\ -x + y = 1,020 \end{cases}$$

Eliminate x .

$$\begin{array}{r} x + y = 1,422 \\ -x + y = 1,020 \\ \hline 2y = 2,442 \\ \frac{2y}{2} = \frac{2,442}{2} \\ y = 1,221 \end{array}$$

$$x + y = 1,422 \text{ The 1}^{\text{st}} \text{ equation}$$

$$x + 1,221 = 1,422$$

$$x + 1,221 - 1,221 = 1,422 - 1,221$$

$$x = 201$$

State the Conclusion

201 miles of the highway is in Alaska.

1,221 miles of the highway is in Canada.

Check the Results

The results check.

19. GOVERNMENT

Analyze the Problem

- Both salaries total \$608,100 a year.
- The president makes \$191,900 more than VP.
- Find the salary of each.

Form the System

Let x = president's salary (larger salary)

y = vice-president salary (smaller salary)

The salary of the President	plus	the salary of the VP	is	\$608,100.
x	+	y	=	608,100
The President	makes	\$191,900	more than	the VP.
x	=	191,900	+	y

Solve the System

$$\begin{cases} x + y = 608,100 \\ x = 191,900 + y \end{cases}$$

Write the 2nd equation in standard form

$$\begin{cases} x + y = 608,100 \\ x - y = 191,900 \end{cases}$$

Eliminate y .

$$\begin{array}{r} x + y = 608,100 \\ x - y = 191,900 \\ \hline 2x = 800,000 \\ \frac{2x}{2} = \frac{800,000}{2} \\ x = 400,000 \end{array}$$

$$\begin{array}{r} x + y = 608,100 \\ \mathbf{400,000} + y = 608,100 \\ 400,000 + y - \mathbf{400,000} = 608,100 - \mathbf{400,000} \\ y = 208,100 \end{array}$$

State the Conclusion

The President's salary is \$400,000.

The Vice-President's salary is \$208,100.

Check the Results

$$\begin{array}{l|l} 400,000 + 208,100 = 608,100 & 400,000 = 191,900 + 208,100 \\ 608,100 = 608,100 & 400,000 = 400,000 \end{array}$$

The results check.

20. CAUSES OF DEATH

Analyze the Problem

- Deaths from both cause is 763,000.
- The number of heart disease deaths is 6 times the number of accident deaths.
- Find the number of deaths from each causes.

Form the System

Let x = # died from heart disease (larger #)

y = # died from accidents (smaller #)

The number of deaths from heart disease	plus	the number of deaths from accidents	is	763,000.
x	+	y	=	763,000
The number of heart deaths	is	6	times	the number of accident deaths.
x	=	6	□	y

Solve the System

$$\begin{cases} x + y = 763,000 \\ x = 6y \end{cases}$$

Use substitution.

$$\begin{array}{r} x + y = 763,000 \text{ The 1}^{\text{st}} \text{ equation} \\ \mathbf{6y} + y = 763,000 \\ 7y = 763,000 \\ \frac{7y}{7} = \frac{763,000}{7} \\ y = 109,000 \end{array}$$

$x = 6y$ The 2nd equation

$$x = 6(\mathbf{109,000})$$

$$x = 654,000$$

State the Conclusion

654,000 people died from heart disease.

109,000 people died from accidents.

Check the Results

The results check.

GEOMETRY PROBLEMS

21. MONUMENTS

Analyze the Problem

- Two angles are supplementary (180°).
- The measure of $\angle 2$ is 15° less than twice the measure of $\angle 1$.
- Find the measure of each angle.

Form the System

Let x = the measure of $\angle 1$ (larger \angle)

y = the measure of $\angle 2$ (smaller \angle)

The measure of $\angle 1$ plus the measure of the $\angle 2$ is supplementary.

$$x + y = 180$$

The measure of $\angle 1$ is 15° less than twice the measure of $\angle 2$.

$$x = 2y - 15$$

Solve the System

$$\begin{cases} x + y = 180 \\ x = 2y - 15 \end{cases}$$

$$x + y = 180 \quad \text{The 1}^{\text{st}} \text{ equation}$$

Substitute for x .

$$(2y - 15) + y = 180$$

$$3y - 15 = 180$$

$$3y - 15 + 15 = 180 + 15$$

$$\frac{3y}{3} = \frac{195}{3}$$

$$y = 65$$

$$x = 2y - 15 \quad \text{The 2}^{\text{nd}} \text{ equation}$$

$$x = 2(65) - 15$$

$$x = 130 - 15$$

$$x = 115$$

State the Conclusion

The measure of $\angle 1$ is 115° .

The measure of $\angle 2$ is 65° .

Check the Results

$$115 + 65 = 180 \quad \left| \quad 115 = 2(65) - 15$$

$$180 = 180 \quad \left| \quad 115 = 130 - 15$$

$$115 = 115$$

The results check.

22. PHYSICAL THERAPY

Analyze the Problem

- Two angles are complementary (90°).
- The current \angle is four times the improvement \angle .
- Find the measure of each angle.

Form the System

Let x = current range of motion (larger \angle)

y = degree of improvement to go (smaller \angle)

The measure of one angle plus the measure of the other angle is complementary.

$$x + y = 90$$

The current angle is four times the improvement angle.

$$x = 4y$$

Solve the System

$$\begin{cases} x + y = 90 \\ x = 4y \end{cases}$$

$$x + y = 90 \quad \text{The 1}^{\text{st}} \text{ equation}$$

Substitute for x .

$$4y + y = 90$$

$$5y = 90$$

$$\frac{5y}{5} = \frac{90}{5}$$

$$y = 18$$

$$x = 4y \quad \text{The 2}^{\text{nd}} \text{ equation}$$

$$x = 4(18)$$

$$x = 72$$

State the Conclusion

The measure of the current angle is 72° .

The measure of the improvement angle is 18° .

Check the Results

The results check.

23. THEATER SCREENS

Analyze the Problem

- A giant rectangular movie screen has a width 26 feet less than its length.
- Its perimeter is 332 feet.
- Find the measures of the width and length.

Form the System

Let w = width of screen in ft (shorter length)

l = length of screen in ft (longer length)

The measure of two widths	plus	the measure of two lengths	is	the perimeter of 332 feet.
$2w$	+	$2l$	=	332

The width	is	26 ft	less than	the length.
w	=	l	-	26

Solve the System

$$\begin{cases} 2l + 2w = 332 \\ w = l - 26 \end{cases}$$

$$2l + 2w = 332 \quad \begin{array}{l} \text{The 1}^{\text{st}} \text{ equation} \\ \text{Substitute for } w. \end{array}$$

$$2l + 2(l - 26) = 332$$

$$2l + 2l - 52 = 332$$

$$4l - 52 = 332$$

$$4l - 52 + 52 = 332 + 52$$

$$4l = 384$$

$$\frac{4l}{4} = \frac{384}{4}$$

$$l = 96$$

$$w = l - 26 \quad \text{The 2}^{\text{nd}} \text{ equation}$$

$$w = 96 - 26$$

$$w = 70$$

State the Conclusion

The width of the screen is 70 feet.

The length of the screen is 96 feet.

Check the Results

$2(70) + 2(96) = 332$	$70 = 96 - 26$
$140 + 192 = 332$	$70 = 70$
$332 = 332$	

The results check.

24. ART

Analyze the Problem

- The sum of the length and width of the painting is 118 inches.
- The difference of the length and width is 22 inches.
- Find the measures of the width and length.

Form the System

Let w = width of painting in inches (shorter length)

l = length of painting in inches (longer length)

The measure of one width	plus	the measure of one length	is	the total of 118 inches.
w	+	l	=	118

The length	minus	the width	is	22 inches.
l	-	w	=	22

Solve the System

$$\begin{cases} l + w = 118 \\ l - w = 22 \end{cases}$$

Eliminate w .

$$l + w = 118$$

$$\frac{l - w = 22}{2l} = 140$$

$$\frac{2l}{2} = \frac{140}{2}$$

$$l = 70$$

$$l + w = 118$$

$$70 + w = 118$$

$$70 + w - 70 = 118 - 70$$

$$w = 48$$

State the Conclusion

The width of the painting is 48 inches.

The length of the painting is 70 inches.

Check the Results

The results check.

25. GEOMETRY

Analyze the Problem

- A 50-meter path surrounds a rectangular garden.
- The width of the garden is two-thirds its length.
- Find the measures of the width and length.

Form the System

Let w = width of garden in m (shorter length)

l = length of garden in m (longer length)

The measure of two widths	plus	the measure of two lengths	is	the perimeter of 50 meters.
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$$2w + 2l = 50$$

The width	is	two-thirds	of	the length.
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$$w = \frac{2}{3}l$$

Solve the System

$$\begin{cases} 2l + 2w = 50 \\ w = \frac{2}{3}l \end{cases}$$

Clear the fraction from the 2nd equation by multiplying both sides by 3.

$$\begin{cases} 2l + 2w = 50 \\ 2l = 3w \end{cases}$$

$$2l + 2w = 50 \quad \begin{array}{l} \text{The 1}^{\text{st}} \text{ equation} \\ \text{Substitute for } 2l. \end{array}$$

$$3w + 2w = 50$$

$$5w = 50$$

$$\frac{5w}{5} = \frac{50}{5}$$

$$w = 10$$

$$2l + 2w = 50 \quad \text{The 1}^{\text{st}} \text{ equation}$$

$$2l + 2(10) = 50$$

$$2l + 20 = 50$$

$$2l + 20 - 20 = 50 - 20$$

$$2l = 30$$

$$\frac{2l}{2} = \frac{30}{2}$$

$$l = 15$$

State the Conclusion

The width of the garden is 10 m.

The length of the garden is 15 m.

Check the Results

$$2(15) + 2(10) = 50 \quad \left| \quad 3(10) = 2(15)$$

$$30 + 20 = 50 \quad \left| \quad 30 = 30$$

$$50 = 50$$

The results check.

26. BALLROOM DANCING

Analyze the Problem

- A rectangular-shaped floor has a perimeter of 200 feet.
- If the floor were 20 feet wider, its width would equal its length.
- Find the measures of the width and length.

Form the System

Let w = width of path in meters (shorter length)

l = length of path in meters (longer length)

The measure of two widths	plus	the measure of two lengths	is	the perimeter of 200 meters.
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$$2w + 2l = 200$$

The length	is	20 feet	wider than	the width.
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$$l = 20 + w$$

Solve the System

$$\begin{cases} 2l + 2w = 200 \\ l = w + 20 \end{cases}$$

$$2l + 2w = 200 \quad \begin{array}{l} \text{The 1}^{\text{st}} \text{ equation} \\ \text{Substitute for } l. \end{array}$$

$$2(w + 20) + 2w = 200$$

$$2w + 40 + 2w = 200$$

$$4w + 40 = 200$$

$$4w + 40 - 40 = 200 - 40$$

$$4w = 160$$

$$\frac{4w}{4} = \frac{160}{4}$$

$$w = 40$$

$$l = w + 20 \quad \text{The 2}^{\text{nd}} \text{ equation}$$

$$l = 40 + 20$$

$$l = 60$$

State the Conclusion

The width of the floor is 40 feet.

The length of the floor is 60 feet.

Check the Results

The results check.

NUMBER/VALUE PROBLEMS

27. EMPTY CARTRIDGES

Analyze the Problem

- \$40 for 5 printer and 2 copier cartridges.
- \$57 for 6 printer and 3 copier cartridges.
- How much is each type of cartridge?

Form the System

Let x = value for a printer cartridge

y = value for a copier cartridge

Number □ Value = Total Value

Printer	5	x	$5x$
Copier	2	y	$2y$

Total 40

The value of 5 printer cartridges	plus	the value of 2 copier cartridges	is	\$40.
$5x$	+	$2y$	=	40

Number □ Value = Total Value

Printer	6	x	$6x$
Copier	3	y	$3y$

Total 57

The value of 6 printer cartridges	plus	the value of 3 copier cartridges	is	\$57.
$6x$	+	$3y$	=	57

Solve the System

$$\begin{cases} 5x + 2y = 40 \\ 6x + 3y = 57 \end{cases}$$

Eliminate y .

Multiply both sides of the 1st equation by 3.

Multiply both sides of the 2nd equation by -2 .

$$15x + 6y = 120$$

$$-12x - 6y = -114$$

$$\hline 3x = 6$$

$$\frac{3x}{3} = \frac{6}{3}$$

$$x = 2$$

$$5x + 2y = 40 \quad \text{The 1st equation}$$

$$5(2) + 2y = 40$$

$$10 + 2y - 10 = 40 - 10$$

$$2y = 30$$

$$\frac{2y}{2} = \frac{30}{2}$$

$$y = 15$$

State the Conclusion

Bank was paid \$2 for each printer cartridge.

Bank was paid \$15 for each copier cartridge.

Check the Results

$$5(\$2) + 2(\$15) = \$40$$

$$\$10 + \$30 = \$40$$

$$\$40 = \$40$$

$$6(\$2) + 3(\$15) = \$57$$

$$\$12 + \$45 = \$57$$

$$\$57 = \$57$$

The results check.

28. THANKSGIVING DINNER

Analyze the Problem

- 510 calories in 6 oz of turkey; 1 slice of pie.
- 580 calories in 4 oz of turkey; 2 slices of pie.
- How many calories are in each?

Form the System

Let x = number of calories in 1 oz of turkey

y = number of calories in 1 slice of pie

Number • Calories = Total Calories

Turkey	6	x	$6x$
Pie	1	y	y
		Total	510

The number of calories in 6 oz of turkey	plus	the number of calories in 1 slice of pie	is	510 calories.
$6x$	+	y	=	510

Number • Calories = Total Calories

Turkey	4	x	$4x$
Pie	2	y	$2y$
		Total	580

The number of calories in 4 oz of turkey	plus	the number of calories in 2 slices of pie	is	580 calories.
$4x$	+	$2y$	=	580

Solve the System

$$\begin{cases} 6x + y = 510 \\ 4x + 2y = 580 \end{cases}$$

Eliminate y .

Multiply both sides of the 1st equation by -2 .

$$-12x - 2y = -1,020$$

$$\hline 4x + 2y = 580$$

$$-8x = -440$$

$$\frac{-8x}{-8} = \frac{-440}{-8}$$

$$x = 55$$

$$6x + y = 510 \quad \text{The 1st equation}$$

$$6(55) + y = 510$$

$$330 + y = 510$$

$$330 + y - 330 = 510 - 330$$

$$y = 180$$

State the Conclusion

55 calories are in one ounce of turkey.

180 calories are in one slice of pumpkin pie.

Check the Results

The results check.

29. WEDDING PICTURES

Analyze the Problem

- Pk 1: 1 10×14 and 10 8×10 photos cost \$239.50.
- Pk 2: 1 10×14 and 5 8×10 photos cost \$134.50.
- Find the cost of each size photo.

Form the System

Let x = the cost of one 10×14 photo

y = the cost of one 8×10 photo

	Number	Value	= Total Value
10×14	1	x	x
8×10	10	y	$10y$

Total **239.50**

The cost of one 10×14 photo plus the cost of ten 8×10 photos is \$239.50.

$$x + 10y = 239.50$$

	Number	Value	= Total Value
10×14	1	x	x
8×10	5	y	$5y$

Total **134.50**

The cost of one 10×14 photo plus the cost of five 8×10 photos is \$134.50.

$$x + 5y = 134.50$$

Solve the System

$$\begin{cases} x + 10y = 239.50 \\ x + 5y = 134.50 \end{cases}$$

Eliminate y .

Multiply both sides of the 2nd equation by -1 .

$$\begin{array}{r} x + 10y = 239.50 \\ -x - 5y = -134.50 \\ \hline \end{array}$$

$$5y = 105$$

$$\frac{5y}{5} = \frac{105}{5}$$

$$y = 21$$

$$x + 5y = 134.50 \text{ The 2}^{\text{nd}} \text{ equation}$$

$$x + 5(21) = 134.50$$

$$x + 105 - 105 = 134.50 - 105$$

$$x = 29.50$$

State the Conclusion

The cost of one 10×14 photo is \$29.50.

The cost of one 8×10 photo is \$21.

Final cost: $\$239.50 + \$29.50 + \$21 = \290

Check the Results

$$\begin{array}{l} \$29.50 + 10(\$21) = \$239.50 \\ \$29.50 + \$210 = \$239.50 \\ \$239.50 = \$239.50 \end{array} \quad \begin{array}{l} \$29.50 + 5(\$21) = \$134.50 \\ \$29.50 + \$105 = \$134.50 \\ \$134.50 = \$134.50 \end{array}$$

The results check.

30. BUYING PAINTING SUPPLIES

Analyze the Problem

- 8 gallons of paint and 3 brushes cost \$270.00.
- 6 gallons of paint and 2 brushes cost \$200.00.
- Find the cost of one gallon of paint and the cost of one brush.

Form the System

Let x = the cost of one gallon of paint

y = the cost of one brush

	Number	Value	= Total Value
Paint	8	x	$8x$
Brushes	3	y	$3y$

Total **270**

The cost of 8 gallons of paint plus the cost of 3 brushes is \$270.00.

$$8x + 3y = 270$$

	Number	Value	= Total Value
Paint	6	x	$6x$
Brushes	2	y	$2y$

Total **200**

The cost of 6 gallons of paint plus the cost of 2 brushes is \$200.00.

$$6x + 2y = 200$$

Solve the System

$$\begin{cases} 8x + 3y = 270 \\ 6x + 2y = 200 \end{cases}$$

Eliminate y .

Multiply both sides of the 1st equation by -2 .

Multiply both sides of the 2nd equation by 3.

$$-16x - 6y = -540$$

$$18x + 6y = 600$$

$$\begin{array}{r} 2x = 60 \\ \hline \end{array}$$

$$\frac{2x}{2} = \frac{60}{2}$$

$$x = 30$$

$$6x + 2y = 200 \text{ The 2}^{\text{nd}} \text{ equation}$$

$$6(30) + 2y = 200$$

$$180 + 2y - 180 = 200 - 180$$

$$2y = 20$$

$$\frac{2y}{2} = \frac{20}{2}$$

$$y = 10$$

State the Conclusion

The cost of one gallon of paint is \$30.

The cost of one brush is \$10.

Check the Results

The results check.

31. COLLECTING STAMPS

Analyze the Problem

- One Elvis stamp and one Lucy stamp cost a total of \$0.63.
- A sheet of 40 Elvis stamps and a sheet of 20 Lucy stamps cost a total of \$18.40.
- How much is each stamp?

Form the System

Let x = the cost of one Elvis stamp

y = the cost of one Lucy stamp

Number □ Value = Total Value

Elvis	1	x	x
Lucy	1	y	y

Total 0.63

The value of one Elvis stamp plus the value of one Lucy stamp is \$0.63.

$$x + y = 0.63$$

Number □ Value = Total Value

Elvis	40	x	$40x$
Lucy	20	y	$20y$

Total 18.40

The value of 40 Elvis stamps plus the value of 20 Lucy stamps is \$18.40.

$$40x + 20y = 18.40$$

Solve the System

$$\begin{cases} x + y = 0.63 \\ 40x + 20y = 18.40 \end{cases}$$

Eliminate y .

Multiply both sides of the 1st equation by -20 .

$$-20x - 20y = -12.60$$

$$40x + 20y = 18.40$$

$$20x = 5.80$$

$$\frac{20x}{20} = \frac{5.80}{20}$$

$$x = 0.29$$

$$x + y = 0.63 \text{ The 1}^{\text{st}} \text{ equation}$$

$$(0.29) + y = 0.63$$

$$0.29 + y - 0.29 = 0.63 - 0.29$$

$$y = 0.34$$

State the Conclusion

The cost of one Elvis stamp is \$0.29.

The cost of one Lucy stamp is \$0.34.

Check the Results

$$\begin{array}{l|l} \$0.29 + \$0.34 = \$0.63 & 40(\$0.29) + 20(\$0.34) = \$18.40 \\ \$0.63 = \$0.63 & \$11.60 + \$6.80 = \$18.40 \\ & \$18.40 = \$18.40 \end{array}$$

The results check.

32. RECYCLING

Analyze the Problem

- Troop earned \$24 by recycling a total of 330 beverage containers.
- Small containers are worth \$0.05 each.
- Large containers are worth \$0.10 each.
- How many of each containers were recycled?

Form the System

Let x = the number of small containers

y = the number of large containers

The number of small containers plus the number of large containers equals the total of 330.

$$x + y = 330$$

Number □ Value = Total Value

Small	x	0.05	$0.05x$
Large	y	0.10	$0.10y$

Total 24

The value of the small containers plus the value of the large containers equals the total value of \$24.00.

$$0.05x + 0.10y = 24$$

Solve the System

$$\begin{cases} x + y = 330 \\ 0.05x + 0.10y = 24 \end{cases}$$

Eliminate x .

Multiply both sides of the 1st equation by -5 .

Multiply both sides of the 2nd equation by 100.

$$-5x - 5y = -1,650$$

$$5x + 10y = 2,400$$

$$5y = 750$$

$$\frac{5y}{5} = \frac{750}{5}$$

$$y = 150$$

$$x + y = 330 \text{ The 1}^{\text{st}} \text{ equation}$$

$$x + (150) = 330$$

$$x + 150 - 150 = 330 - 150$$

$$x = 180$$

State the Conclusion

The quantity of small containers is 180.

The quantity of large containers is 150.

Check the Results

The results check.

33. SELLING ICE CREAM

Analyze the Problem

- Ice cream cones cost \$1.80 and sundaes cost \$3.30.
- The receipts for a total of 148 cones and sundaes were \$360.90.
- How many of each was sold?

Form the System

Let x = number of cones sold
 y = number of sundaes sold

The number of cones	plus	the number of sundaes	is	148.
x	+	y	=	148

Number • Value = Total Value

Cones	x	1.80	1.80x
Sundaes	y	3.30	3.30y
		Total	360.90

The value of x cones	plus	the value of y sundaes	is	\$360.90.
$1.80x$	+	$3.30y$	=	360.90

Solve the System

$$\begin{cases} x + y = 148 \\ 1.80x + 3.30y = 360.90 \end{cases}$$

Solve the 1st equation for x .

$$\begin{cases} x = 148 - y \\ 1.80x + 3.30y = 360.90 \end{cases}$$

$$1.80x + 3.30y = 360.90 \quad \begin{array}{l} \text{The 2}^{\text{nd}} \text{ equation} \\ \text{Substitute for } x. \end{array}$$

$$1.80(148 - y) + 3.30y = 360.90$$

$$266.40 - 1.80y + 3.30y = 360.90$$

$$266.40 + 1.50y = 360.90$$

$$266.40 + 1.50y - 266.40 = 360.90 - 266.40$$

$$1.50y = 94.50$$

$$\frac{1.50y}{1.50} = \frac{94.50}{1.50}$$

$$\mathbf{1.50} \quad \mathbf{1.50}$$

$$y = 63$$

$$x + y = 148 \quad \text{The 1}^{\text{st}} \text{ equation}$$

$$x + \mathbf{63} = 148$$

$$x + 63 - \mathbf{63} = 148 - \mathbf{63}$$

$$x = 85$$

State the Conclusion

The number of cones sold is 85.

The number of sunades sold is 63.

Check the Results

$$85 + 63 = 148$$

$$148 = 148$$

$$\$1.80(\mathbf{85}) + \$3.30(\mathbf{63}) = \$360.90$$

$$\$153 + \$207.90 = \$360.90$$

$$\$360.90 = \$360.90$$

The results check.

34. BUYING TICKETS

Analyze the Problem

- Receipts were \$1,740 for 190 people.
- General admission tickets cost \$10.00.
- Senior admission tickets cost \$6.00.
- How many of each type were sold?

Form an Equation

Let x = number of general admission tickets
 y = number of senior tickets

The number of gen ad tickets	plus	the number of senior tickets	is	190.
x	+	y	=	190

Number • Value = Total Value

Gen Ad	x	\$10.00	10x
Senior	y	\$6.00	6y
		Total	1,740

The value of general adm	plus	the value of senior	is	\$1,740.
$10x$	+	$6y$	=	1,740

Solve the System

$$\begin{cases} x + y = 190 \\ 10x + 6y = 1,740 \end{cases}$$

Eliminate y .

Multiply both sides of the 1st equation by -6 .

$$-6x - 6y = -1,140$$

$$10x + 6y = 1,740$$

$$4x = 300$$

$$\frac{4x}{4} = \frac{300}{4}$$

$$\mathbf{4} \quad \mathbf{4}$$

$$x = 150$$

$$x + y = 190 \quad \text{The 1}^{\text{st}} \text{ equation}$$

$$\mathbf{150} + y = 190$$

$$150 + y - \mathbf{150} = 190 - \mathbf{150}$$

$$y = 40$$

State the Conclusion

150 general admission tickets were sold.

40 senior citizen tickets were sold.

Check the Results

The results check.

INTEREST PROBLEMS

35. STUDENT LOANS

Analyze the Problem

- A college used \$5,000 to make two student loans.
- The first at 5% annual interest to a nursing student.
- The second at 7% to a business major.
- College collected \$310 in interest the first year.
- How much was loaned to each student?

Form the System

Let x = amount loaned to nursing student

y = amount loaned to business student

The amount loaned to nursing student	plus	the amount loaned to business student	is	\$5,000.
x	+	y	=	5,000
Principal	□	Rate	□	Time = Interest

Nurse	x	5%	1 yr	$0.05x$
Business	y	7%	1 yr	$0.07y$

Total Interest = \$310

Solve the System

$$\begin{cases} x + y = 5,000 \\ 0.05x + 0.07y = 310 \end{cases}$$

Eliminate x .

Multiply both sides of the 1st equation by -5 .

Multiply both sides of the 2nd equation by 100.

$$-5x - 5y = -25,000$$

$$\underline{5x + 7y = 31,000}$$

$$2y = 6,000$$

$$\underline{2y = 6,000}$$

$$\underline{2} \quad \underline{2}$$

$$y = 3,000$$

$$x + y = 5,000 \quad \text{The 1}^{\text{st}} \text{ equation}$$

$$x + \mathbf{3,000} = 5,000$$

$$x + 3,000 - \mathbf{3,000} = 5,000 - \mathbf{3,000}$$

$$x = 2,000$$

State the Conclusion

The nursing student loan is \$2,000.

The business student loan is \$3,000

Check the Results

$\$2,000 + \$3,000 = \$5,000$	$0.05(\$2,000) + 0.07(\$3,000) = \$310$
$\$5,000 = \$5,000$	$\$100 + \$210 = \$310$
	$\$310 = \310

The results check.

36. FINANCIAL PLANNING

Analyze the Problem

- Investing \$6,000.
- Savings account paying 6% annual interest.
- Development plan paying 12% annually.
- Combined interest for first year was \$540.
- How much was invested at each rate?

Form the System

Let x = amount invested in savings account

y = amount invested in mini-mall

The amount invested in savings	plus	the amount invested in the mini-mall	is	\$6,000.
x	+	y	=	6,000
Principal	□	Rate	□	Time = Interest

Savings	x	6%	1 yr	$0.06x$
Mini-mall	y	12%	1 yr	$0.12y$

Total Interest = \$540

The interest earned by savings	plus	the interest earned by mini-mall	is	\$540.
$0.06x$	+	$0.12y$	=	540

Solve the System

$$\begin{cases} x + y = 6,000 \\ 0.06x + 0.12y = 540 \end{cases}$$

Eliminate x .

Multiply both sides of the 1st equation by -6 .

Multiply both sides of the 2nd equation by 100.

$$-6x - 6y = -36,000$$

$$\underline{6x + 12y = 54,000}$$

$$6y = 18,000$$

$$\underline{6y = 18,000}$$

$$\underline{6} \quad \underline{6}$$

$$y = 3,000$$

$$x + y = 6,000 \quad \text{The 1}^{\text{st}} \text{ equation}$$

$$x + \mathbf{3,000} = 6,000$$

$$x + 3,000 - \mathbf{3,000} = 6,000 - \mathbf{3,000}$$

$$x = 3,000$$

State the Conclusion

\$3,000 was invested at 6%.

\$3,000 was invested at 12%.

Check the Results

The results check.

37. INVESTING A BONUS

Analyze the Problem

- Bonus amount is \$40,000.
- Invested part in a fund at 8% annual interest.
- The rest in an offshore bank at 9% annual interest.
- \$3,415 in interest was earned the first year.
- How much was invested at each rate?

Form the System

Let x = amount invested at 8%

y = amount invested at 9%

The amount invested in an int. fund	plus	the amount invested in an offshore bank	is	\$40,000.
x	+	y	=	40,000

Principal □ Rate □ Time = Interest

int fund	x	0.08	1 yr	0.08x
offshore bank	y	0.09	1 yr	0.09y

Total Interest = \$3,415

Solve the System

$$\begin{cases} x + y = 40,000 \\ 0.08x + 0.09y = 3,415 \end{cases}$$

Eliminate x .

Multiply both sides of the 1st equation by -8 .

Multiply both sides of the 2nd equation by 100.

$$\begin{array}{r} -8x - 8y = -320,000 \\ 8x + 9y = 341,500 \\ \hline y = 21,500 \end{array}$$

$$x + y = 40,000 \quad \text{The 1}^{\text{st}} \text{ equation}$$

$$x + 21,500 = 40,000$$

$$x + 21,500 - 21,500 = 40,000 - 21,500$$

$$x = 18,500$$

State the Conclusion

\$18,500 was invested at 8%.

\$21,500 was invested at 9%.

Check the Results

$\$18,500 + \$21,500 = \$40,000$ $\$40,000 = \$40,000$	$0.08(\$18,500) + 0.09(\$21,500) = \$3,415$ $\$1,480 + \$1,935 = \$3,415$ $\$3,415 = \$3,415$
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The results check.

38. PENSION FUNDS

Analyze the Problem

- Pension fund has \$1,000,000 to invest.
- Invested part at 3.5% annual interest.
- The rest at 4.5% annual interest.
- \$39,000 in interest was earned the first year.
- How much was invested at each rate?

Form the System

Let x = amount invested at 3.5%

y = amount invested at 4.5%

The amount invested at 3.5%	plus	the amount invested at 4.5%	is	\$1,000,000.
x	+	y	=	1,000,000

Principal □ Rate □ Time = Interest

First part	x	0.035	1 yr	0.035x
Second part	y	0.045	1 yr	0.045y

Total Interest = \$39,000

Solve the System

$$\begin{cases} x + y = 1,000,000 \\ 0.035x + 0.045y = 39,000 \end{cases}$$

Eliminate x .

Multiply both sides of the 1st equation by -35 .

Multiply both sides of the 2nd equation by 1,000.

$$\begin{array}{r} -35x - 35y = -35,000,000 \\ 35x + 45y = 39,000,000 \\ \hline 10y = 4,000,000 \\ 10y = 4,000,000 \\ \hline \mathbf{10} \qquad \mathbf{10} \\ y = 400,000 \end{array}$$

$$x + y = 1,000,000 \quad \text{The 1}^{\text{st}} \text{ equation}$$

$$x + 400,000 = 1,000,000$$

$$x + 400,000 - 400,000 = 1,000,000 - 400,000$$

$$x = 600,000$$

State the Conclusion

\$400,000 was invested at 3.5%.

\$600,000 was invested at 4.5%.

Check the Results

The results check.

39. LOSSES

Analyze the Problem

- CEO has \$22,000 to invest.
- Invested part at 4% annual interest.
- Invested in biotech at 3% annual interest.
- Biotech showed a loss after first year.
- \$110 in interest was earned the first year.
- Find the amount of each investment.

Form the System

Let x = amount invested at 4%

y = amount invested at 3%

The amount invested in the 4% account	plus	the amount invested in the 3% biotech	is	\$22,000.
x	+	y	=	$22,000$

Principal Rate Time = Interest

account	x	0.04	1 yr	0.04x
biotech	y	0.03	1 yr	-0.03y

Total Interest = \$110

Solve the System

$$\begin{cases} x + y = 22,000 \\ 0.04x - 0.03y = 110 \end{cases}$$

Eliminate y .

Multiply both sides of the 1st equation by 3.

Multiply both sides of the 2nd equation by 100.

$$3x + 3y = 66,000$$

$$4x - 3y = 11,000$$

$$7x = 77,000$$

$$\frac{7x}{7} = \frac{77,000}{7}$$

$$x = 11,000$$

$$x + y = 22,000 \quad \text{The 1}^{\text{st}} \text{ equation}$$

$$x + \mathbf{11,000} = 22,000$$

$$x + 11,000 - \mathbf{11,000} = 22,000 - \mathbf{11,000}$$

$$x = 11,000$$

State the Conclusion

\$11,000 was invested at 4%.

\$11,000 was invested at 3%.

Check the Results

$\$11,000 + \$11,000 = \$22,000$	$0.04(\$11,000) - 0.03(\$11,000) = \$110$
$\$22,000 = \$22,000$	$\$440 - \$330 = \$110$
	$\$110 = \110

The results check.

40. LOTTERY WINNINGS

Analyze the Problem

- Retired teacher won \$60,000.
- Gave \$10,000 to grandchildren.
- Invested in growth fund at 4.4% annual interest.
- Invested in certificates at 5.8% annual interest.
- \$2,732 in interest was earned the first year.
- Find the amount of each investment.

Form the System

Let x = amount invested at 4.4%

y = amount invested at 5.8%

The amount invested in the 4.4% account	plus	the amount invested in the 5.8% account	is	\$50,000.
x	+	y	=	$50,000$

Principal Rate Time = Interest

Growth fund	x	0.044	1 yr	0.044x
certificates	y	0.058	1 yr	0.058y

Total Interest = \$2,732

Solve the System

$$\begin{cases} x + y = 50,000 \\ 0.044x + 0.058y = 2,732 \end{cases}$$

Eliminate x .

Multiply both sides of the 1st equation by -44 .

Multiply both sides of the 2nd equation by 1,000.

$$-44x - 44y = -2,200,000$$

$$44x + 58y = 2,732,000$$

$$14y = 532,000$$

$$\frac{14y}{14} = \frac{532,000}{14}$$

$$\mathbf{14} \quad \mathbf{14}$$

$$y = 38,000$$

$$x + y = 50,000 \quad \text{The 1}^{\text{st}} \text{ equation}$$

$$x + \mathbf{38,000} = 50,000$$

$$x + 38,000 - \mathbf{38,000} = 50,000 - \mathbf{38,000}$$

$$x = 12,000$$

State the Conclusion

\$12,000 was invested at 4.4%.

\$38,000 was invested at 5.8%.

Check the Results

The results check.

UNIFORM MOTION PROBLEMS

41. THE GULF STREAM

Analyze the Problem

- With the current, a ship traveled 300 mi in 10 hrs.
- Against the current, it took 15 hours for return trip.
- Find the speed of the ship in still water and the speed of the current.

Form the System

Let x = speed of ship in still water in mph
 y = speed of current in mph

$$\text{Rate} \cdot \text{Time} = \text{Distance}$$

With current	$x + y$	10	$10(x + y)$
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$$\text{Total Distance} = 300$$

The rate of the ship with the current	times	10 hours traveled	is	300 miles.
$(x + y)$	\square	10	=	300

$$\text{Rate} \cdot \text{Time} = \text{Distance}$$

Against current	$x - y$	15	$15(x - y)$
-----------------	---------	----	-------------

$$\text{Total Distance} = 300$$

The rate of the ship against the current	times	15 hours traveled	is	300 miles.
$(x - y)$	\square	15	=	300

Solve the System

$$\begin{cases} 10(x + y) = 300 \\ 15(x - y) = 300 \end{cases}$$

Distribute in both equations.

$$\begin{cases} 10x + 10y = 300 \\ 15x - 15y = 300 \end{cases}$$

Eliminate y .

Multiply both sides of the 1st equation by 3.

Multiply both sides of the 2nd equation by 2.

$$30x + 30y = 900$$

$$30x - 30y = 600$$

$$\hline 60x \quad \quad = 1,500$$

$$\frac{60x}{60} = \frac{1,500}{60}$$

$$\frac{60}{60} = \frac{25}{1}$$

$$x = 25$$

$$10(x + y) = 300 \quad \text{The 1}^{\text{st}} \text{ equation}$$

$$10(25 + y) = 300$$

$$250 + 10y = 300$$

$$250 + 10y - 250 = 300 - 250$$

$$10y = 50$$

$$\frac{10y}{10} = \frac{50}{10}$$

$$\frac{10}{10} = \frac{5}{1}$$

$$y = 5$$

State the Conclusion

The speed of the ship in still water is 25 mph.

The speed of the current is 5 mph.

Check the Results

$10(25 + 5) = 300$	$15(25 - 5) = 300$
$10(30) = 300$	$15(20) = 300$
$300 = 300$	$300 = 300$

The results check.

42. THE JET STREAM

Analyze the Problem

- Flying with jet stream, plane flew 3,000 mi in 5 hr.
- Against same wind, trip took 6 hours.
- Find the speed of the plane in still air and the speed of the wind current.

Form the System

Let x = speed of plane in still air in mph

y = speed of jet current in mph

$$\text{Rate} \cdot \text{Time} = \text{Distance}$$

With jet stream	$x + y$	5	$5(x + y)$
-----------------	---------	---	------------

$$\text{Total Distance} = 3,000$$

The rate of the plane with the wind	times	5 hours traveled	is	3,000 miles.
$(x + y)$	\square	5	=	3,000

$$\text{Rate} \cdot \text{Time} = \text{Distance}$$

Against jet stream	$x - y$	6	$6(x - y)$
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$$\text{Total Distance} = 3,000$$

The rate of the plane against the wind	times	6 hours traveled	is	3,000 miles.
$(x - y)$	\square	6	=	3,000

Solve the System

$$\begin{cases} 5(x + y) = 3,000 \\ 6(x - y) = 3,000 \end{cases}$$

Distribute in both equations.

$$\begin{cases} 5x + 5y = 3,000 \\ 6x - 6y = 3,000 \end{cases}$$

Eliminate y .

Multiply both sides of the 1st equation by 6.

Multiply both sides of the 2nd equation by 5.

$$30x + 30y = 18,000$$

$$30x - 30y = 15,000$$

$$\hline 60x \quad \quad = 33,000$$

$$\frac{60x}{60} = \frac{33,000}{60}$$

$$\frac{60}{60} = \frac{550}{1}$$

$$x = 550$$

$$5(x + y) = 3,000 \quad \text{The 1}^{\text{st}} \text{ equation}$$

$$5(550 + y) = 3,000$$

$$2,750 + 5y = 3,000$$

$$2,750 + 5y - 2,750 = 3,000 - 2,750$$

$$5y = 250$$

$$\frac{5y}{5} = \frac{250}{5}$$

$$\frac{5}{5} = \frac{50}{1}$$

$$y = 50$$

State the Conclusion

The speed of the plane in still air is 550 mph.

The speed of the jet stream is 50 mph.

Check the Results

The results check.

43. AVIATION

Analyze the Problem

- Airplane flies with the wind 800 mi in 4 hrs.
- The return trip against the wind takes 5 hours.
- Find the speed of the plane in still air and the speed of the wind current.

Form the System

Let x = speed of plane in still air in mph

y = speed of wind in mph

$$\text{Rate} \cdot \text{Time} = \text{Distance}$$

With wind	$x + y$	4	$4(x + y)$
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$$\text{Total Distance} = 800$$

The rate of the plane with the wind times 4 hours traveled is 800 miles.

$$(x + y) \cdot 4 = 800$$

$$\text{Rate} \cdot \text{Time} = \text{Distance}$$

Against wind	$x - y$	5	$5(x - y)$
--------------	---------	---	------------

$$\text{Total Distance} = 800$$

The rate of the plane against the wind times 5 hours traveled is 800 miles.

$$(x - y) \cdot 5 = 800$$

Solve the System

$$\begin{cases} 4(x + y) = 800 \\ 5(x - y) = 800 \end{cases}$$

Distribute in both equations.

$$\begin{cases} 4x + 4y = 800 \\ 5x - 5y = 800 \end{cases}$$

Eliminate y .

Multiply both sides of the 1st equation by 5.

Multiply both sides of the 2nd equation by 4.

$$20x + 20y = 4,000$$

$$20x - 20y = 3,200$$

$$40x = 7,200$$

$$\frac{40x}{40} = \frac{7,200}{40}$$

$$\mathbf{40} \quad \mathbf{40}$$

$$x = 180$$

$$4(x + y) = 800 \quad \text{The 1}^{\text{st}} \text{ equation}$$

$$4(\mathbf{180} + y) = 800$$

$$720 + 4y = 800$$

$$720 + 4y - \mathbf{720} = 800 - \mathbf{720}$$

$$4y = 80$$

$$\frac{4y}{4} = \frac{80}{4}$$

$$\mathbf{4} \quad \mathbf{4}$$

$$y = 20$$

State the Conclusion

The speed of the plane in still air is 180 mph.

The speed of the wind is 20 mph.

Check the Results

$$\begin{array}{l|l} 4(180 + 20) = 800 & 5(180 - 20) = 800 \\ 4(200) = 800 & 5(160) = 800 \\ 800 = 800 & 800 = 800 \end{array}$$

The results check.

44. BOATING

Analyze the Problem

- A boat can travel 24 miles downstream in 2 hours.
- The return trip takes 3 hours.
- Find the speed of the boat in still water and the speed of the current.

Form the System

Let x = speed of boat in still water in mph

y = speed of stream in mph

$$\text{Rate} \cdot \text{Time} = \text{Distance}$$

With current	$x + y$	2	$2(x + y)$
--------------	---------	---	------------

$$\text{Total Distance} = 24$$

The rate of the boat with the current times 2 hours traveled is 24 miles.

$$(x + y) \cdot 2 = 24$$

$$\text{Rate} \cdot \text{Time} = \text{Distance}$$

Against current	$x - y$	3	$3(x - y)$
-----------------	---------	---	------------

$$\text{Total Distance} = 24$$

The rate of the boat against the current times 3 hours traveled is 24 miles.

$$(x - y) \cdot 3 = 24$$

Solve the System

$$\begin{cases} 2(x + y) = 24 \\ 3(x - y) = 24 \end{cases}$$

Distribute in both equations.

$$\begin{cases} 2x + 2y = 24 \\ 3x - 3y = 24 \end{cases}$$

Eliminate y .

Multiply both sides of the 1st equation by 3.

Multiply both sides of the 2nd equation by 2.

$$6x + 6y = 72$$

$$6x - 6y = 48$$

$$12x = 120$$

$$\frac{12x}{12} = \frac{120}{12}$$

$$\mathbf{12} \quad \mathbf{12}$$

$$x = 10$$

$$2(x + y) = 24 \quad \text{The 1}^{\text{st}} \text{ equation}$$

$$2(\mathbf{10} + y) = 24$$

$$20 + 2y = 24$$

$$20 + 2y - \mathbf{20} = 24 - \mathbf{20}$$

$$2y = 4$$

$$\frac{2y}{2} = \frac{4}{2}$$

$$\mathbf{2} \quad \mathbf{2}$$

$$y = 2$$

State the Conclusion

The speed of the boat in still water is 10 mph.

The speed of the current is 2 mph.

Check the Results

The results check.

45. MARINE BIOLOGY

Analyze the Problem

- Set up an aquarium containing 3% salt water.
- 2 tanks contain 6% and 2% salt water.
- How much from each tank must he use to fill a 32-gallon aquarium with a 3% mixture?

Form the System

Let x = amount of 2% salt water in gallons
 y = amount of 6% salt water in gallons

The # of gallons of 2% salt water	plus	the # of gallons of 6% salt water	is	32 gallons.
x	+	y	=	32

Amt • Strength = Amt of salt												
<table style="width: 100%; border-collapse: collapse;"> <tr> <td style="padding: 2px;">Weak</td> <td style="padding: 2px;">x</td> <td style="padding: 2px;">0.02</td> <td style="padding: 2px;">0.02x</td> </tr> <tr> <td style="padding: 2px;">Strong</td> <td style="padding: 2px;">y</td> <td style="padding: 2px;">0.06</td> <td style="padding: 2px;">0.06y</td> </tr> <tr> <td style="padding: 2px;">Mix</td> <td style="padding: 2px;">32</td> <td style="padding: 2px;">0.03</td> <td style="padding: 2px;">32(0.03)</td> </tr> </table>	Weak	x	0.02	0.02x	Strong	y	0.06	0.06y	Mix	32	0.03	32(0.03)
Weak	x	0.02	0.02x									
Strong	y	0.06	0.06y									
Mix	32	0.03	32(0.03)									

The amount of salt in the 2% salt water	plus	the amount of salt in the 6% salt water	is equal to	the amount of salt in the 3% salt water.
$0.02x$	+	$0.06y$	=	$32(0.03)$

Solve the System

$$\begin{cases} x + y = 32 \\ 0.02x + 0.06y = 0.96 \end{cases}$$

Eliminate x .

Multiply both sides of the 1st equation by -2 .

Multiply both sides of the 2nd equation by 100.

$$\begin{array}{r} -2x - 2y = -64 \\ 2x + 6y = 96 \\ \hline 4y = 32 \\ \frac{4y}{4} = \frac{32}{4} \\ y = 8 \end{array}$$

$$x + y = 32 \text{ The 1}^{\text{st}} \text{ equation}$$

$$x + 8 = 32$$

$$x + 8 - 8 = 32 - 8$$

$$x = 24$$

State the Conclusion

24 gallons of 2% salt water will be needed.
 8 gallons of 6% salt water will be needed.

Check the Results

$24 + 8 = 32$	$0.02(24) + 0.06(8) = 0.96$
$32 = 32$	$0.48 + 0.48 = 0.96$
	$0.96 = 0.96$

The results check.

46. COMMEMORATIVE COINS

Analyze the Problem

- Coins to be made from a 40% silver alloy.
- Foundry has two alloys, one with 50% silver content and one with a 25% silver content.
- How many kilograms of each alloy should be used to make 20 kg of the 40% silver alloy?

Form the System

Let x = amount of 25% silver alloy in kgs
 y = amount of 50% silver alloy in kgs

The # of kg of 25% alloy	plus	the # of kg of 50% alloy	is	20 kilograms.
x	+	y	=	20

Amt • Strength = Amt of silver												
<table style="width: 100%; border-collapse: collapse;"> <tr> <td style="padding: 2px;">Less silver</td> <td style="padding: 2px;">x</td> <td style="padding: 2px;">0.25</td> <td style="padding: 2px;">0.25x</td> </tr> <tr> <td style="padding: 2px;">More silver</td> <td style="padding: 2px;">y</td> <td style="padding: 2px;">0.50</td> <td style="padding: 2px;">0.50y</td> </tr> <tr> <td style="padding: 2px;">Mix</td> <td style="padding: 2px;">20</td> <td style="padding: 2px;">0.40</td> <td style="padding: 2px;">20(0.40)</td> </tr> </table>	Less silver	x	0.25	0.25x	More silver	y	0.50	0.50y	Mix	20	0.40	20(0.40)
Less silver	x	0.25	0.25x									
More silver	y	0.50	0.50y									
Mix	20	0.40	20(0.40)									

The amount of silver in the 25% alloy	plus	the amount of silver in the 50% alloy	is equal to	the amount of silver in the 40% alloy.
$0.25x$	+	$0.50y$	=	$20(0.40)$

Solve the System

$$\begin{cases} x + y = 20 \\ 0.25x + 0.50y = 8 \end{cases}$$

Eliminate x .

Multiply both sides of the 1st equation by -25 .

Multiply both sides of the 2nd equation by 100.

$$\begin{array}{r} -25x - 25y = -500 \\ 25x + 50y = 800 \\ \hline 25y = 300 \\ \frac{25y}{25} = \frac{300}{25} \\ y = 12 \end{array}$$

$$x + y = 20 \text{ The 1}^{\text{st}} \text{ equation}$$

$$x + 12 = 20$$

$$x + 12 - 12 = 20 - 12$$

$$x = 8$$

State the Conclusion

8 kg of 25% silver alloy will be needed.
 12 kg of 50% silver alloy will be needed.

Check the Results

The results check.

47. CLEANING FLOORS

Analyze the Problem

- Custodian mixes a 4% ammonia solution and a 12% ammonia solution to get 1 gallon (128 fluid ounces) of a 9% ammonia solution.
- How many fluid ounces of the 4% solution and the 12% solution should be used?

Form the System

Let x = amount of 4% ammonia solution in oz
 y = amount of 12% ammonia solution in oz

The # of ounces of 4% ammonia	plus	the # of ounces of 12% ammonia	is	128 ounces.
x	+	y	=	128

Amt • Strength = Amt of salt

Weak	x	0.04	0.04x
Strong	y	0.12	0.12y
Mix	128	0.09	128(0.09)

The amount of ammonia in the 4% solution	plus	the amount of ammonia in the 12% solution	is equal to	the amount of ammonia in the 9% solution.
$0.04x$	+	$0.12y$	=	$128(0.09)$

Solve the System

$$\begin{cases} x + y = 128 \\ 0.04x + 0.12y = 11.52 \end{cases}$$

Eliminate x .

Multiply both sides of the 1st equation by -4 .

Multiply both sides of the 2nd equation by 100 .

$$\begin{array}{r} -4x - 4y = -512 \\ 4x + 12y = 1,152 \\ \hline 8y = 640 \\ 8y = 640 \\ \hline 8 \quad 8 \\ y = 80 \end{array}$$

$$x + y = 128 \quad \text{The 1}^{\text{st}} \text{ equation}$$

$$x + 80 = 128$$

$$x + 80 - 80 = 128 - 80$$

$$x = 48$$

State the Conclusion

48 ounces of 4% solution will be needed.
 80 ounces of 12% solution will be needed.

Check the Results

$48 + 80 = 128$	$0.04(48) + 0.12(80) = 11.52$
$128 = 128$	$1.92 + 9.6 = 11.52$
	$11.52 = 11.52$

The results check.

48. MOUTHWASH

Analyze the Problem

- 2 mouthwash solutions: 6% ethanol alcohol and 18% ethanol alcohol.
- How many mL of each must be mixed to make 750 mL that is 10% ethanol alcohol?

Form the System

Let x = amount of 6% mouthwash in mL
 y = amount of 18% mouthwash in mL

The # of ml of 6% mouthwash	plus	the # of ml of 18% mouthwash	is	750 milliliters.
x	+	y	=	750

Amt • Strength = Amt of alcohol

Weak	x	0.06	0.06x
Strong	y	0.18	0.18y
Mix	750	0.10	750(0.10)

The amount of alcohol in the 6% solution	plus	the amount of alcohol in the 18% solution	is equal to	the amount of alcohol in the 10% solution.
$0.06x$	+	$0.18y$	=	$750(0.10)$

Solve the System

$$\begin{cases} x + y = 750 \\ 0.06x + 0.18y = 7.5 \end{cases}$$

Eliminate x .

Multiply both sides of the 1st equation by -6 .

Multiply both sides of the 2nd equation by 100 .

$$\begin{array}{r} -6x - 6y = -4,500 \\ 6x + 18y = 7,500 \\ \hline 12y = 3,000 \\ 12y = 3,000 \\ \hline 12 \quad 12 \\ y = 250 \end{array}$$

$$x + y = 750 \quad \text{The 1}^{\text{st}} \text{ equation}$$

$$x + 250 = 750$$

$$x + 250 - 250 = 750 - 250$$

$$x = 500$$

State the Conclusion

500 mL of 6% mouth wash will be needed.
 250 mL of 18% mouth wash will be needed.

Check the Results

The results check.

49. COFFEE SALES

Analyze the Problem

- A 100 pound blend sells for \$6.35 a pound.
- Brazilian coffee selling for \$3.75 a pound.
- Columbian coffee selling for \$8.75 a pound.
- How much of each type should be used?

Form the System

Let x = amount of \$8.75/lb coffee in lbs

y = amount of \$3.75/lb coffee in lbs

The # of lbs of \$8.75/lb coffee	plus	the # of lbs of \$3.75/lb coffee	is	100 pounds.
x	+	y	=	100
		Amt • Cost/lb =	Total Value	

Columbian	x	8.75	8.75x
Brazilian	y	3.75	3.75y
Mixture	100	6.35	100(6.35)

The value of \$8.75/lb coffee	plus	the value of \$3.75/lb coffee	is equal to	the value of \$6.35/lb coffee.
$8.75x$	+	$3.75y$	=	$6.35(100)$

Solve the System

$$\begin{cases} x + y = 100 \\ 8.75x + 3.75y = 635 \end{cases}$$

Eliminate y .

Multiply both sides of the 1st equation by -375 .

Multiply both sides of the 2nd equation by 100 .

$$\begin{array}{r} -375x - 375y = -37,500 \\ 875x + 375y = 63,500 \\ \hline 500x = 26,000 \\ \frac{500x}{500} = \frac{26,000}{500} \\ x = 52 \end{array}$$

$$x + y = 100 \text{ The 1}^{\text{st}} \text{ equation}$$

$$52 + y = 100$$

$$52 + y - 52 = 100 - 52$$

$$y = 48$$

State the Conclusion

52 lb of \$8.75/lb coffee will be needed.

48 lb of \$3.75/lb coffee will be needed.

Check the Results

$52 + 48 = 100$	$\$8.75(52) + \$3.75(48) = \$635$
$100 = 100$	$\$455 + \$180 = \$635$
	$\$635 = \635

The results check.

50. MIXING NUTS

Analyze the Problem

- Merchant mixes peanuts with cashews to get 48 pounds of mixed nuts.
- Mixture sells at \$8 per pound.
- Peanuts sell for \$6.00 a pound.
- Cashews sell for \$12.00 a pound.
- How much of each type should be used?

Form the System

Let x = amount of \$6.00/lb peanuts in lbs

y = amount of \$12.00/lb cashews in lbs

The # of lbs of \$6.00/lb peanuts	plus	the # of lbs of \$12.00/lb cashews	is	48 pounds.
x	+	y	=	48
		Amt • Cost/lb =	Total Value	

Peanuts	x	6.00	6.00x
Cashews	y	12.00	12.00y
Mixture	48	8.00	8.00(48)

The value of \$6/lb peanuts	plus	the value of \$12/lb cashews	is equal to	the value of \$8/lb blend.
$6x$	+	$12y$	=	$8(48)$

Solve the System

$$\begin{cases} x + y = 48 \\ 6x + 12y = 384 \end{cases}$$

Eliminate x .

Multiply both sides of the 1st equation by -6 .

$$\begin{array}{r} -6x - 6y = -288 \\ 6x + 12y = 384 \\ \hline 6y = 96 \\ \frac{6y}{6} = \frac{96}{6} \\ y = 16 \end{array}$$

$$x + y = 48 \text{ The 1}^{\text{st}} \text{ equation}$$

$$x + 16 = 48$$

$$x + 16 - 16 = 48 - 16$$

$$x = 32$$

State the Conclusion

32 lb of \$6.00/lb peanuts will be needed.

16 lb of \$12.00/lb cashews will be needed.

Check the Results

The results check.

51. GOURMET FOODS

Analyze the Problem

- Stuffed Kalamata olives sell for \$9 a pint.
- Marinated mushrooms sell for \$12 a pint.
- How many pints of each are needed to get 20 pints of a mixture that will sell for \$10 a pint?

Form the System

Let x = pints of \$9.00/pt olives
 y = pints of \$12.00/pt mushrooms

The # of pints of \$9.00/pt olives	plus	the # of pints of \$12.00/pt mushrooms	is	a total of 20 pints.
x	+	y	=	20

Amt • Cost/pt = Total Value

Olives	x	9.00	9.00x
Mushrooms	y	12.00	12.00y
Mixture	20	10.00	10.00(20)

The value of \$9/pt olives	plus	the value of \$12/pt mushrooms	is equal to	the value of \$10/pt blend.
$9x$	+	$12y$	=	$10(20)$

Solve the System

$$\begin{cases} x + y = 20 \\ 9x + 12y = 200 \end{cases}$$

Eliminate x .

Multiply both sides of the 1st equation by -9 .

$$\begin{array}{r} -9x - 9y = -180 \\ 9x + 12y = 200 \\ \hline \end{array}$$

$$3y = 20$$

$$\frac{3y}{3} = \frac{20}{3}$$

$$y = 2\frac{1}{2}$$

$$x + y = 20 \quad \text{The 1}^{\text{st}} \text{ equation}$$

$$x + 6\frac{2}{3} = 20$$

$$x + 6\frac{2}{3} - 6\frac{2}{3} = 19\frac{3}{3} - 6\frac{2}{3}$$

$$x = 13\frac{1}{3}$$

State the Conclusion

$13\frac{1}{3}$ pt of \$9.00/pt olives will be needed.

$6\frac{2}{3}$ pt of \$12.00/pt mushrooms will be needed.

Check the Results

$13\frac{1}{3} + 6\frac{2}{3} = 20$ $20 = 20$	$\$9\left(\frac{40}{3}\right) + \$12\left(\frac{20}{3}\right) = \200 $\$120 + \$80 = \$200$ $\$200 = \200
--	---

The results check.

52. HERBS

Analyze the Problem

- Ginger root powder sells for \$6.50/lb.
- Ginkgo leaf powder sells for \$9.50/lb.
- How many lbs of each are needed to make 15 lbs of a mixture that sells for \$7/lb?

Form the System

Let x = lbs of \$6.50/lb ginger root
 y = lbs of \$9.50/lb ginkgo leaf

The # of lbs of \$6.50/lb ginger	plus	the # of lbs of \$9.50/lb ginkgo	is	a total of 15 pounds.
x	+	y	=	15

Amt • Cost/lb = Total Value

Ginger Root	x	6.50	6.50x
Ginkgo Leaf	y	9.50	9.50y
Mixture	15	7.00	7.00(15)

The value of \$6.50/lb ginger	plus	the value of \$9.50/lb ginkgo	is equal to	the value of \$7/lb blend.
$6.50x$	+	$9.50y$	=	$7.00(15)$

Solve the System

$$\begin{cases} x + y = 15 \\ 6.50x + 9.50y = 105 \end{cases}$$

Eliminate x .

Multiply both sides of the 1st equation by -650 .

Multiply both sides of the 2nd equation by 100.

$$\begin{array}{r} -650x - 650y = -9,750 \\ 650x + 950y = 10,500 \\ \hline \end{array}$$

$$300y = 750$$

$$\frac{300y}{300} = \frac{750}{300}$$

$$y = 2\frac{1}{2}$$

$$x + y = 15 \quad \text{The 1}^{\text{st}} \text{ equation}$$

$$x + 2\frac{1}{2} = 15$$

$$x + 2\frac{1}{2} - 2\frac{1}{2} = 14\frac{2}{2} - 2\frac{1}{2}$$

$$x = 12\frac{1}{2}$$

State the Conclusion

$12\frac{1}{2}$ lb of \$6.50/lb ginger root will be needed.

$2\frac{1}{2}$ lb of \$9.50/lb ginkgo leaf will be needed.

Check the Results

The results check.

WRITING

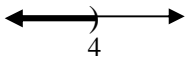
53. Answers will vary.

54. Answers will vary.

REVIEW

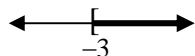
Graph each inequality. Then describe the graph using interval notation.

55. $x < 4$



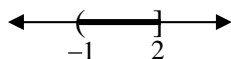
$(-\infty, 4)$

56. $x \geq -3$



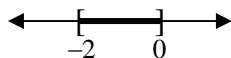
$[-3, \infty)$

57. $-1 < x \leq 2$



$(-1, 2]$

58. $-2 \leq x \leq 0$



$[-2, 0]$

CHALLENGE PROBLEMS

59. SCALES

Analyze the Problem

3 nails and 1 bolt is the same as 3 nuts.

1 bolt and 1 nut is the same as 5 nails.

How many nails will it take to balance 1 nut?

Form the System

Three nails

 plus

one bolt

 is

3 nuts.

3 nails + 1 bolt = 3 nuts

One bolt

 plus

one nut

 is

5 nails.

1 bolt + 1 nut = 5 nails

Solve the System

$\begin{cases} 3 \text{ nails} + 1 \text{ bolt} = 3 \text{ nuts} \\ 1 \text{ bolt} + 1 \text{ nut} = 5 \text{ nails} \end{cases}$

$\begin{cases} 3 \text{ nails} + 1 \text{ bolt} = 3 \text{ nuts} \\ 1 \text{ bolt} = 5 \text{ nails} - 1 \text{ nut} \end{cases}$

Solve the 2nd equation for 1 bolt.

$\begin{cases} 3 \text{ nails} + 1 \text{ bolt} = 3 \text{ nuts} \\ 1 \text{ bolt} = 5 \text{ nails} - 1 \text{ nut} \end{cases}$

$3 \text{ nails} + (5 \text{ nails} - 1 \text{ nut}) = 3 \text{ nuts}$

$3 \text{ nails} + 1 \text{ bolt} = 3 \text{ nuts}$ The 1st equation
Substitute for 1 bolt.

$3 \text{ nails} + (5 \text{ nails} - 1 \text{ nut}) = 3 \text{ nuts}$

$8 \text{ nails} - 1 \text{ nut} = 3 \text{ nuts}$

$8 \text{ nails} - 1 \text{ nut} + 1 \text{ nut} = 3 \text{ nuts} + 1 \text{ nut}$

$8 \text{ nails} = 4 \text{ nuts}$

$\frac{8 \text{ nails}}{4} = \frac{4 \text{ nuts}}{4}$

$2 \text{ nails} = 1 \text{ nut}$

State the Conclusion

It will take 2 nails to balance 1 nut.

Check the Results

The results check.

60. FARMING

Analyze the Problem

Goats and chickens.

There are 40 heads and 130 feet.

How many goats and chickens are there?

Form the System

Let g = # of goat heads

c = # of chicken heads

$4g$ = # of goat feet

$2c$ = # of chicken feet

The # of goat heads

 plus

the # of chicken heads

 is

40 heads.

$g + c = 40$

The # of goat feet

 plus

the # of chicken feet

 is

130 feet.

$4g + 2c = 130$

Solve the System

$\begin{cases} g + c = 40 \\ 4g + 2c = 130 \end{cases}$

Eliminate c .

Multiply both sides of the 1st equation by -2 .

$-2g - 2c = -80$

$4g + 2c = 130$

$2g = 50$

$\frac{2g}{2} = \frac{50}{2}$

$g = 25$

$g + c = 40$ The 1st equation

$25 + c = 40$

$25 + c - 25 = 40 - 25$

$c = 15$

State the Conclusion

There are 25 goats.

There are 15 chickens.

Check the Results

The results check.